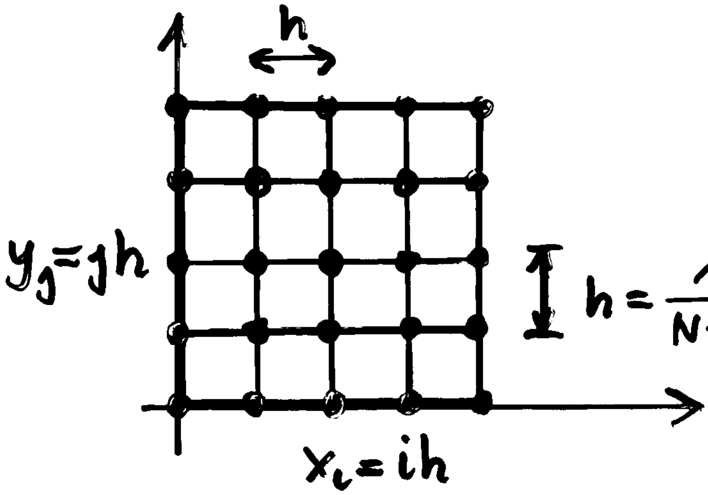
Finite-Differenzen-Diskretisierung:

- $\overline{\Omega} \longmapsto \overline{\Omega}_h = \{(x_i, y_j) = (ih, jh) : i,j = 0,1,\dots,N_h+1\}$



$h = \frac{1}{N+1}$, $N = N_h$

- Ableitungen $\longmapsto$ Differenzen:

$$\frac{\partial^2 u}{\partial x^2}(x_i, y_j) = \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right)(x_i, y_j)$$

$$\approx \frac{1}{h}\left[\frac{\partial u}{\partial x}\left(x_i + \frac{h}{2}, y_j\right) - \frac{\partial u}{\partial x}\left(x_i - \frac{h}{2}, y_j\right)\right]$$

$$\approx \frac{1}{h}\left[\frac{u(x_{i+1}, y_j) - u(x_i, y_j)}{h} - \frac{u(x_i, y_j) - u(x_{i-1}, y_j)}{h}\right]$$

$$\approx \frac{1}{h^2}\left[u_{i+1,j} - 2u_{ij} + u_{i-1,j}\right] =: u_{\bar{x}x}(x_i, y_j)$$

$u_{ij} \approx u(x_i, y_j)$

Analog:

$$\frac{\partial^2 u}{\partial y^2}(x_i, y_j) \approx \frac{1}{h^2}\left[u_{i,j+1} - 2u_{ij} + u_{i,j-1}\right] =: u_{y\bar{y}}(x_i, y_j)$$

$f_{ij} := f(x_i, y_j)$