T U T O R I A L

"Numerical Methods for Solving Partial Differential Equations"

to the Lectures on NuPDE

T IX Monday, 8 January 2007 (Time: 13:45 - 15:15, Room: P 004)

1.9 FEM for BVPs of Second-order ODEs

44 Consider the elliptic problem: find $u(x)$ such that

$$\begin{aligned} -u''(x) &= 8 \qquad x \in (0,1), \\ u(x) &= -1 \qquad x = 0, \\ u(x) + u'(x) &= -5 \qquad x = 1. \end{aligned} \tag{1.23}$$

1. Write the variational formulation of (1.23):

$$\text{find } u \in V_0 : \quad a(u,v) = \langle F, v \rangle \qquad \forall v \in V_0 \, . \tag{1.24}$$

Specify the functional space V_0, the definition of the bilinear form $a(\cdot,\cdot)$ and of the linear functional F.

2. Consider a finite element discretization of (1.24) based on Courant elements, to give the linear system

$$K_h \underline{u}_h = \underline{f}_h \, . \tag{1.25}$$

Suppose to use first a grid with $h_1 = 0.1$ and then another finer grid with $h_2 = 0.05$. Solve the linear system (1.25) by the preconditioned conjugate gradient method, adopting the following preconditioning strategies:

a) without preconditioner ($C = I$);
b) with Jacobi preconditioner $C = \text{diag}(K_h)$;
c) with MDS preconditioner with 2 grids $\mathcal{T}_1$, $\mathcal{T}_2$ ($L = 2$);
d) with MDS preconditioner with 4 grids $\mathcal{T}_1, \ldots, \mathcal{T}_4$ ($L = 4$).

Fill in the following tables:

Number of iterations

	$C = I$	Jacobi	MDS ($L = 2$)	MDS ($L = 4$)
$h_1 = 0.1$				
$h_2 = 0.05$				

CPU time

	$C = I$	Jacobi	MDS ($L = 2$)	MDS ($L = 4$)
$h_1 = 0.1$				
$h_2 = 0.05$				

How do the number of iterations and the CPU time depend on the mesh size h and on the number of unknowns?