T U T O R I A L

"Numerical Methods for Solving Partial Differential Equations"

to the Lectures on NuPDE

T VII Monday, 4 December 2006 (Time: 13:45 - 15:15, Room: P 004)

1.7 FEM for BVPs of Second-order ODEs

34 Consider the linear system $Ku = f$ where K is a symmetric and positive definite matrix. Let $r^{(n)}$ be the residual of the approximation $u^{(n)}$ of u and let $p^{(n)}$ denote a search direction at the step n. Given $p^{(n-1)}$, determine the real parameter $\beta^{(n)} \in \mathbb{R}$, such that the search direction $p^{(n)}$

$$p^{(n)} = r^{(n)} + \beta^{(n-1)} p^{(n-1)}$$

satisfies

$$(Kp^{(n)}, Kp^{(n-1)})_{\ell_2} = 0.$$

35 Consider the linear system $Ku = f$ where K is a symmetric and positive definite matrix. The so-called *conjugate residual (CR) method* reads:

given $u^{(0)}$, compute the residual $r^{(0)} = f - Ku^{(0)}$. Then, for $n \geq 0$,

$$\begin{aligned} p^{(n)} &= \begin{cases} r^{(0)} & \text{if } n = 0 \\ r^{(n)} + \beta^{(n-1)} p^{(n-1)} & \text{if } n \geq 1 \end{cases} \\ u^{(n+1)} &= u^{(n)} + \alpha^{(n)} p^{(n)} \\ r^{(n+1)} &= r^{(n)} - \alpha^{(n)} K p^{(n)}. \end{aligned}$$

The parameters $\beta^{(n-1)}$ and $\alpha^{(n)}$ are chosen such that

$$(Kp^{(n)}, Kp^{(n-1)})_{\ell_2} = 0$$

and

$$\|f - Ku^{(n+1)}\|_{\ell_2} = \min_{v \in u^{(n)} + \text{span}(p^{(n)})} \|f - Kv\|_{\ell_2}.$$

Prove that

$$(Kr^{(n+1)}, p^{(n)})_{\ell_2} = 0 \quad \text{and} \quad (Kr^{(n+1)}, r^{(n)})_{\ell_2} = 0.$$

36 Consider the linear system

$$Ku = f, \tag{1.21}$$

where K is a symmetric and positive definite matrix. Write the conjugate gradient (CG) method to solve (1.21) using the scalar product

$$(w, v)_K = (Kw, v)_{\ell_2} \tag{1.22}$$

instead of the Euclidean scalar product $(w, v)_{\ell_2}$, and show that it coincides with the CR method.