2. Newmark-Method for $(2)_c$ $Mu'' + Cu' + Ku = f$:

- **Motivation:** $u(t)$

$$v(t) = u'(t)$$
$$a(t) = u''(t) = v'(t)$$
$$u(t_{j+1}) = u(t_j) + v(t_j)\tau + a(t_j + \theta\tau)\frac{\tau^2}{2}$$
$$v(t_{j+1}) = v(t_j) + a(t_j + \tilde{\theta}\tau)\tau$$
with $\theta, \tilde{\theta} \in (0,1)$

- **Newmark-Family:** $\beta \in [0, 1/2]$, $\gamma \in [0,1]$

(4)

Find $a^{j+1} \in \mathbb{R}^{N_h}$:

$$M a^{j+1} + C(t_{j+1}) v^{j+1} + K(t_{j+1}) u^{j+1} = f^{j+1} := f_h(t_{j+1}),$$
$$u^{j+1} = u^j + \tau v^j + \frac{\tau^2}{2}\{(1-2\beta) a^j + 2\beta a^{j+1}\}$$
$$v^{j+1} = v^j + \tau\{(1-\gamma) a^j + \gamma a^{j+1}\}$$
$j = 0, 1, 2, \dots, m-1$

IC: $u^0 = M_h^{-1} \underline{u}_{0h}$, $v^0 = M_h^{-1} \underline{v}_{0h}$ given, $a^0 = ?$

$$(4) \quad (M + \tau\gamma C(t_{j+1}) + \tau^2 \beta K(t_{j+1}))\, a^{j+1} = RHS^j$$

$$M a^0 + C(t_0) v^0 + K(t_0) u^0 = f^0$$

- The case $\boxed{\beta = \frac{1}{4} \text{ and } \gamma = \frac{1}{2}}$ corresponds to the TR = Crank-Nicolson

(mms)

applied to (3) $\begin{pmatrix} u(t) \\ v(t) \end{pmatrix}' = \begin{pmatrix} v(t) \\ f(t, u(t), v(t)) \end{pmatrix}$

$(u(0), v(0)) = (u_0, v_0)$