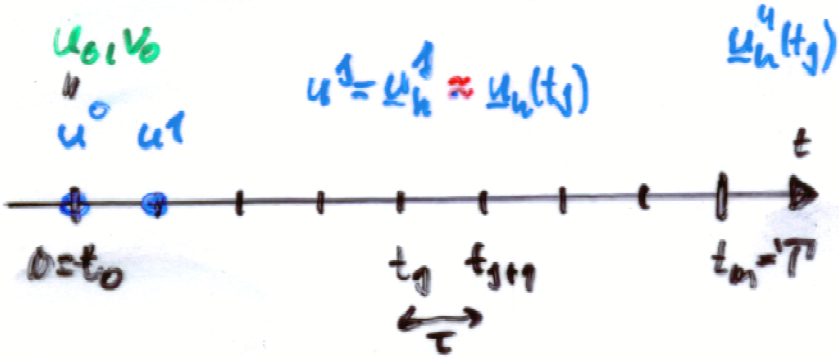
Discretization in Time:

1. σ-Method (= Finite Difference Method) for (2) $Mu'' + Ku = f$ + IC:



$$\underline{u}_h''(t_j) \approx u_{\tau t}^j = \frac{u^{j+1} - 2u^j + u^{j-1}}{\tau^2}$$

(3) Find $u^{j+1} = \underline{u}_h^{j+1} \in \mathbb{R}^{N_h}$: $\quad \sigma \in [0,1]$

$$M u_{\tau t}^j + \sigma K(t_{j+1}) u^{j+1} + (1-2\sigma) K(t_j) u^j + \sigma K(t_{j-1}) u^{j-1} = f_\sigma^j \stackrel{\text{def}}{:=} \sigma \underline{f}_h(t_{j+1}) + (1-2\sigma) \underline{f}_h(t_j) + \sigma \underline{f}_h(t_{j-1})$$

$j = 1, 2, \dots, m-1$; u^0, u^1 - given (mass)

- Consistency order = $O(\tau^2)$
- Stability: $\sigma \geq \frac{1}{4} \Rightarrow$ unconditionally stable
  $\sigma = 0 \Rightarrow$ CFL-condition
  $$\tau < \frac{2}{\sqrt{\lambda_{max}(M_h^{-1} K_h)}}$$
- Error $= O(h^2 + \tau^2)$
- (3) means solving the system

$$(M + \tau^2 \sigma K) u^{j+1} = RHS^j, \quad j = \overline{1, m-1}$$
u^0, u^1 given