

(2)_c

$$\text{Find } \underline{u}_h(t) : M_h \underline{u}_h''(t) + \underline{C}_h(t) \underline{u}_h'(t) + \underline{K}_h(t) \underline{u}_h(t) = \underline{f}_h(t) \quad \forall t \in (0, T) \\ M_h \underline{u}_h(0) = \underline{u}_{0h}, \quad M_h \underline{u}_h'(0) = \underline{v}_{0h}$$



2nd order ODE system in $\mathbb{R}^{N_h}$

$$\text{Find } u(t) = \underline{u}_h(t) : u''(t) = f(t, u(t), u'(t)), t \in (0, T) \\ u(0) = u_0 \\ u'(0) = v_0$$



with

$$f(t, u(t), u'(t)) := M_h^{-1} (\underline{f}_h(t) - \underline{K}_h(t) \underline{u}_h(t) - \underline{C}_h(t) \underline{u}_h'(t)) \\ u_0 = M_h^{-1} \underline{u}_{0h}, \quad v_0 = M_h^{-1} \underline{v}_{0h}$$

$$v(t) = u'(t)$$

(3)

$$\text{Find } U(t) := \begin{pmatrix} u(t) \\ v(t) \end{pmatrix} : [0, T] \rightarrow \mathbb{R}^{n=2N_h}$$

$$U'(t) = F(t, U(t))$$

$$U(0) = U_0$$

$$\begin{pmatrix} u(t) \\ v(t) \end{pmatrix}' = \begin{pmatrix} v(t) \\ f(t, u(t), v(t)) \end{pmatrix}$$

$$\begin{pmatrix} u(0) \\ v(0) \end{pmatrix} = \begin{pmatrix} u_0 \\ v_0 \end{pmatrix}$$

= 1st order ODE system in $\mathbb{R}^{2N_h}$