- **Result:** RKF applied to (53) for a normal matrix J is contractive iff $|R(\tau\lambda)| \le 1 \;\forall \lambda \in \sigma(J)$.

- **Example 2.28:** cf. also Example 2.20

  Consider again $(\underline{6})_h$ $\underline{u}_h'(t) = J\,\underline{u}_h(t) + M_h^{-1}\underline{f}_h(t)$ with $J = -M_h^{-1}K_h = J^*$ wrt $(\cdot,\cdot) = (\cdot,\cdot)_{M_h}$, where M_h, K_h are SPD, i.e.

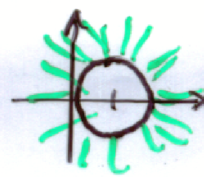
  $$\lambda_{min}(J) = -\lambda_{max}(M_h^{-1}K_h) \le \lambda(J) \le -\lambda_{min}(M_h^{-1}K_h) < 0,$$

  i.e. $(\underline{6})_h$ is dissipative.

  - Explicit Euler is contractive iff

$\frac{h^2}{6}$

$$\|R(\tau J)\| = \max_{\lambda\in\sigma(J)} |R(\tau\lambda)| \le 1 \overset{\uparrow}{\underset{R(z)=1+z}{\Leftrightarrow}} \tau \le \frac{2}{\lambda_{max}(M_h^{-1}K_h)} = O(h^2)$$

  - Implicit Euler: = A-stable := $\mathbb{C}^- \subset S$ = 

  $$\underbrace{\|R(\tau J)\| = \max_{\lambda\in\sigma(J)}}_{\mathbb{R}^-} |R(\tau\lambda)| \underset{R(z)=\frac{1}{1-z}}{\le} 1 \quad \forall \tau > 0$$

- In general, for an A-stable (implicit) RKF applied to a dissipative system $u' = Ju + f$ with a normal coefficient matrix J, the condition

  $$|R(\lambda\tau)| \le 1 \quad \forall \lambda \in \sigma(J) \subset \mathbb{C}^-$$

  is always satisfied. Therefore, the method is <u>contractive</u>.

- Surprisingly, this is also true for non-normal matrices J as follows from Theorem 2.29.