- J normal $\Rightarrow \exists$ unitary matrix U (i.e. $U^*U = UU^* = I$):

$$J = UDU^* \quad \text{i.e.} \quad JU = UD$$

$$D = \operatorname{diag}(\lambda_1, \lambda_2, \dots, \lambda_n), \quad U = [u_1, \dots, u_n]$$

eigenvalues — eigenvectors (ONS)

- System (53) is dissipative iff $\forall v \in \mathbb{C}^n$

$$\operatorname{Re}(Jv, v) = (J \operatorname{Re} v, \operatorname{Re} v) + (J \operatorname{Im} v, \operatorname{Im} v) \le 0.$$

- Now, $\forall v \in \mathbb{C}^n$, we have

$$\operatorname{Re}(Jv, v) = \operatorname{Re}(UDU^*v, v) = \operatorname{Re}(D\underbrace{U^*v}_{w}, \underbrace{U^*v}_{w})$$

$$= \operatorname{Re}(Dw, w) =$$

$$= ((\operatorname{Re} D)\operatorname{Re} w, \operatorname{Re} w) + ((\operatorname{Re} D)\operatorname{Im} w, \operatorname{Im} w)$$

$$= \sum_{i=1}^{n} \operatorname{Re}\lambda_i |w_i|^2$$

$v = \sum_{i=1}^{n} \alpha_i u_i$ (ON)

- This immediately implies that the system (53) is dissipative iff

$$\sum_{i=1}^{n} \operatorname{Re}\lambda_i |w_i|^2 \le 0 \ \forall w \in \mathbb{C}^n, \ \text{i.e.} \ \operatorname{Re}\lambda_i \le 0 \ \forall i = \overline{1,n}.$$

q.e.d ∎

- <u>Lemma 2.27:</u>

For normal matrices J, we have

$$(58) \qquad \|R(\tau J)\| = \max_{\lambda \in \sigma(J)} |R(\tau\lambda)|.$$

<u>Proof:</u> $v = v_1 + i v_2 \in \mathbb{C}$ — sups $\mathbb{R}^n, \mathbb{C}^n \dots \setminus \{0\}$

$$\|R(\tau J)\|^2 = \sup_{w \in \mathbb{R}^n \setminus \{0\}} \frac{\|R(\tau J)w\|^2}{\|w\|^2} =$$

$$= \sup_{v_1, v_2 \in \mathbb{R}^n} \frac{\|R(\tau J)v_1\|^2 + \|R(\tau J)v_2\|^2}{\|v_1\|^2 + \|v_2\|^2} = \sup_{v \in \mathbb{C}} \frac{\|R(\tau J)v\|^2}{\|v\|^2}$$

$$= \sup_{\alpha} \frac{\sum_i |\alpha_i|^2 |R(\tau\lambda_i)|^2}{\sum_i |\alpha_i|^2} = \max_i |R(\tau\lambda_i)|^2.$$

$$v = \sum_{i=1}^{n} \alpha_i u_i, \quad \|v\|^2 = (v, v) = \sum_i |\alpha_i|^2$$

$$R(\tau J)v = \sum_i \alpha_i R(\tau\lambda_i) u_i, \quad \|R(\tau J)v\|^2 = \sum_i |\alpha_i|^2 |R(\tau\lambda_i)|^2$$

q.e.d. ∎