

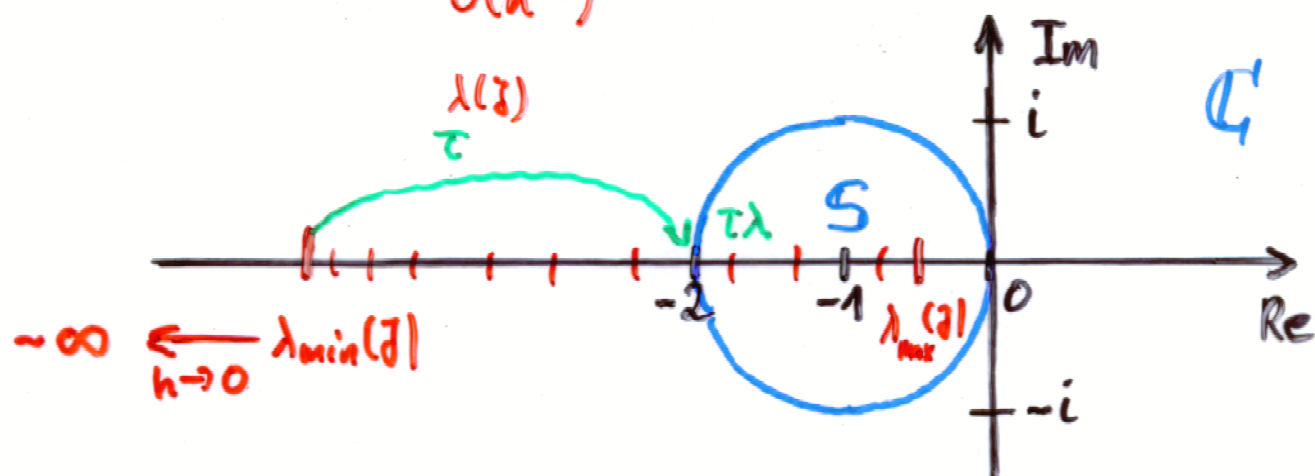
• Example 2.20:

The VML applied to our model problem (3) leads to

(6)_h $u_h'(t) = J u_h(t) + M_h^{-1} f_h(t)$ with $J = -M_h^{-1} K_h$.

K_h and M_h are SPD, i.e. $\sigma(J) \subset \mathbb{R}^-$;

$-\infty \xleftarrow{h \rightarrow 0} \lambda_{\min}(J) = -\lambda_{\max}(M_h^{-1} K_h) \leq \dots \leq \lambda_{\max}(J) = -\lambda_{\min}(M_h^{-1} K_h) < 0$
 $O(h^{-2})$



Stability domain for the explicit Euler method:

$$S = \{z \in \mathbb{C} : |z - (-1)| \leq 1\} =$$

A small diagram showing the stability domain S as a yellow-shaded circle in the complex plane, centered at -1 on the real axis with a radius of 1. The axes are labeled 'Re' and 'Im'.

Thus, $\tau \lambda \in S \forall \lambda \in \sigma(J)$ means

$$\tau \lambda_{\min}(J) = -\tau \lambda_{\max}(M_h^{-1} K_h) \geq -2$$

(44)

$$\tau \leq \frac{2}{\lambda_{\max}(M_h^{-1} K_h)}$$

Since $\lambda_{\max}(M_h^{-1} K_h) \leq 12 h^{-2}$ (see (28)), condition (43) is satisfied if

$$\tau \leq h^2 / 6.$$

This is a strong restriction on the time step size τ , in particular, if the spatial step size h is small!