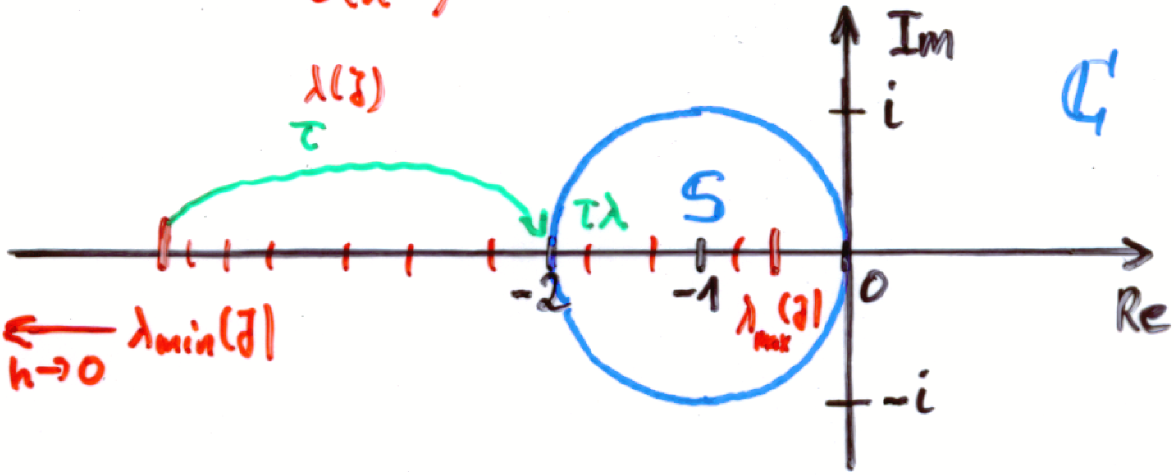
• Example 2.20:

The VML applied to our model problem (3) leads to

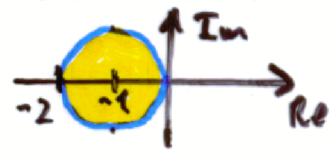
$$(6)_h \quad \underline{u}_h'(t) = J\,\underline{u}_h(t) + M_h^{-1}\underline{f}_h(t) \quad \text{with } J = -M_h^{-1}K_h .$$

K_h and M_h are SPD, i.e. $\sigma(J) \subset \mathbb{R}^-$:

$$-\infty \xleftarrow[h\to 0]{} \lambda_{min}(J) = -\underbrace{\lambda_{max}(M_h^{-1}K_h)}_{O(h^{-2})} \le \ldots \le \lambda_{max}(J) = -\lambda_{min}(M_h^{-1}K_h) < 0$$



Stability domain for the explicit Euler method:

$$S = \{ z \in \mathbb{C} : |z-(-1)| \le 1 \} =$$ 

Thus, $\tau\lambda \in S \;\forall \lambda \in \sigma(J)$ means

$$\tau\,\lambda_{min}(J) = -\tau\,\lambda_{max}(M_h^{-1}K_h) \ge -2$$

$$\Updownarrow$$

$$(44) \qquad \tau \le \frac{2}{\lambda_{max}(M_h^{-1}K_h)}$$

Since $\lambda_{max}(M_h^{-1}K_h) \le 12\,h^{-2}$ (see (28)), condition (43) is satisfied if

$$\tau \le h^2/6 .$$

This is a strong restriction on the time step size τ, in particular, if the spatial step size h is small!