$\Longrightarrow$ Therefore, the error $e(t) = \tilde{u}(t) - u(t)$ must satisfy

$$e'(t) = \underbrace{f_u(t, u(t))}_{\substack{\text{Freeze the Jacobi matrix} \\ J = f_u(t_*, u(t_*)) \in \mathbb{C}^{n \times n},\ \text{e.g. at } t_* = 0.}} e(t) + \cancel{o(e(t))}\ , \quad e(0) = \delta$$

$\Longrightarrow$ The ODE

$$(38) \qquad e'(t) = J e(t) \qquad (\text{cf. } (6)_h : J = -M_h^{-1} K_h)$$

is a <u>good model</u> for the error propagation.

$\Longrightarrow$ Assume that $J = X D X^{-1}$, $D = \operatorname{diag}[\lambda_1, \dots, \lambda_n]$, $X = [x_1, x_2, \dots, x_n]$, is diagonalizable, i.e.

- $\exists$ complete basis of EV: $x_1, x_2, \dots, x_n \in \mathbb{C}^n$
- corresponding eigenvalues : $\lambda_1, \lambda_2, \dots, \lambda_n \in \mathbb{C}$.

Then, inserting the ansatz

$$e(t) = \sum_{i=1}^{n} \eta_i(t)\, x_i$$

into (38), we get

$$e'(t) \equiv \sum_{i=1}^{n} \eta_i'(t)\, x_i = J e(t) \equiv \sum_{i=1}^{n} \eta_i(t)\, \lambda_i x_i,$$

i.e.

$$\boxed{\eta_i'(t) = \lambda\, \eta_i(t) \quad \forall i = \overline{1,n}} \ \widehat{=}\ (37)$$

$\Longrightarrow$ <u>Result</u>:

The behavior of the time integration method (RKF) for (37) with $\lambda \in \sigma(f_u(t, u(t))) := \{\text{eigenvalues of the Jacobi-matrix}\}$ reflects the behavior of the time integration method in the general case (15)!