Remark 2.17:
To investigate the stability behavior of time integration methods (here: RKF), we apply the method to the test problem (37) with $\lambda \in \mathbb{C}$:

(37) $\boxed{\begin{aligned} u'(t) &= \lambda u(t) \\ u(0) &= 1 \end{aligned}}$ $\rightsquigarrow$ $\boxed{\begin{aligned} \tilde{u}'(t) &= \lambda \tilde{u}(t) \\ \tilde{u}(0) &= 1 + \delta \end{aligned}}$ $(\widetilde{37})$ $\delta > 0$

$$u(t) = e^{\lambda t} \qquad \tilde{u}(t) = (1+\delta)\, e^{\lambda t}$$

error propagation

$\lambda = \operatorname{Re}\lambda + i \operatorname{Im}\lambda$

$$e(t) = \tilde{u}(t) - u(t) = \delta e^{\lambda t} = \delta\, e^{\operatorname{Re}\lambda\, t}\, e^{i \operatorname{Im}\lambda\, t}$$

$$\Longrightarrow \boxed{|e(t)| = \delta e^{\operatorname{Re}\lambda\, t} \le \delta \quad \text{if } \operatorname{Re}\lambda \le 0} \iff \text{dissipative}$$

OK, but will we get information about the general case $u'(t) = f(t, u(t))$ from studing RKF applied to (37)?

YES, as the following investigation shows:

(15) $\begin{Bmatrix} u'(t) = f(t, u(t)) \\ u(0) = u_0 \end{Bmatrix}$ $\curvearrowright$ $u(t)$ – exact solution

$(\widetilde{15})$ $\begin{Bmatrix} \tilde{u}'(t) = f(t, \tilde{u}(t)) \\ \tilde{u}(0) = u_0 + \delta \end{Bmatrix}$ $\curvearrowright$ $\tilde{u}(t)$ – perturbed solution

Linearization of the perturbed problem $(\widetilde{15})$ in the neighbourhood of the exact solution $u(t)$ gives:

$$\Longrightarrow \tilde{u}'(t) = f(t, u(t) + (\tilde{u}(t) - u(t))) =$$
$$= \underbrace{f(t, u(t))}_{= u'(t)} + f_u(t, u(t)) \underbrace{(\tilde{u}(t) - u(t))}_{=: e(t)} + o(\tilde{u}(t) - u(t))$$