Discretization Error: $\|u_h(t) - u(t)\|_H \le ?$

- <u>Definition 2.8:</u> Ritz projection = elliptic projection
  Let $a(\cdot,\cdot): V\times V \to \mathbb{R}$ be a V-elliptic (coercive) and V-bounded (contineous) bilinear form on V.
  The Ritz projection $R_h : V \longrightarrow V_h$ is defined by the identity
  (11) $\quad a(R_h w, v_h) = a(w, v_h) \;\forall v_h \in V_h \subset V, \;\forall w \in V.$

- The Ritz projection $R_h \in L(V, V_h)$ describes the approximate solution $u_h \in V_h$ of the VP
  $$a(u, v) = \langle F, v\rangle \;\forall v \in V$$
  by the Galerkin method in the form
  $$u_h = R_h u.$$

- Then Cea's theorem (Lemma) reads as follows:
  $$\|w - R_h w\|_V \le \frac{\mu_2}{\mu_1} \inf_{w_h \in V_h} \|w - w_h\|_V .$$

- <u>Definition 2.9:</u> H-projection $\quad P_h = Q_h$
  The projection $P_h : H \to V_h$ is defined by the identity
  (12) $\quad (P_h w, v_h)_H = (w, v_h)_H \;\forall v_h \in V_h \;\forall w \in H.$

  For $H = L_2(\Omega)$, the operator P_h is called L_2-projection on V_h.

- For the initial condition, we have
  $$(u_h(0), v_h)_H = (u_0, v_h)_H = (P_h u_0, v_h)_H \;\forall v_h \in V_h,$$
  and, therefore,
  $$u_h(0) = P_h u_0 =: u_{0h} \in V_h .$$