A-priori Estimates:

From the estimate

$$\|u_h(0)\|_H^2 = (u_h(0), u_h(0))_H = (u_0, u_h(0))_H \le \|u_0\|_H \|u_h(0)\|_H$$
$$\Longrightarrow \|u_h(0)\|_H \le \|u_0\|_H$$

and the proof of Lemma 2.6, we immetiately get the a-priori estimates (uniformly in h !)

$$(10) \quad \|u_h\|_X \le c_1, \quad \|Au_h\|_X \le c_2, \quad \|u_h(t)\|_h \le c_3 \quad \forall h \in$$

where $X = H^1((0,T), V)$ and c_i from Lemma 2.6.

Existence of a solution of the continuous problem:

- At the first glance:
  Apply Picard-Lindelöf proof to

$$(6) \quad \boxed{\begin{array}{l} u'(t) + Au(t) = F(t) \quad \forall t \in (0,T) \\ u(0) = u_0 \end{array}}$$

  (it looks like (9)!)
  But this fails ! Why ? (moes")
- However, the $\exists!$ $\underline{u}_h(t) \leftrightarrow u_h(t) \in H^1((0,T), V_h)$: $\underline{(6)}_h$
  and the a-priori bounds ensure the existence of the solution $u \in H^1((0,T), V)$: (6).
  $\Rightarrow$ Theorem 2.7 : see T13 !
- Theorem 2.7. + Lemma 2.5: $\exists!$