- **Remark 1.28:** $n = I(\varepsilon)$

$n = I(\varepsilon)$ = number of iterations for reducing the initial error by the factor $\varepsilon = 10^{-t} \in (0,1)$, i.e.

$$\|\underline{u}_h - \underline{u}_h^n\|_{C_h} \le q^n \|\underline{u}_h - \underline{u}_h^0\|_{C_h} \le \varepsilon \|\underline{u}_h - \underline{u}_h^0\|_{C_h}$$

$$\boxed{q^n \le \varepsilon}$$

$$\Updownarrow$$

$$n = I(\varepsilon) = \left[\!\left[ \frac{\ln \varepsilon^{-1}}{\ln q^{-1}} \right]\!\right] := \text{entire}(\ldots) + 1$$

If $\nu_2/\nu_1 \gg 1$ (e.g. our example: $\nu_2/\nu_1 = O(h^{-2})$), then we have

$$I(\varepsilon) = \frac{\ln \varepsilon^{-1}}{\ln q^{-1}} = \begin{cases} \dfrac{\ln \varepsilon^{-1}}{-\ln\sqrt{1-(\nu_1/\nu_2)^2}} \approx 2 \ln \varepsilon^{-1} \left(\dfrac{\nu_2}{\nu_1}\right)^2 & (N) \\[2ex] \dfrac{\ln \varepsilon^{-1}}{-\ln\left(1 - 2/\left(\frac{\nu_2}{\nu_1} - 1\right)\right)} \approx \dfrac{1}{2} \ln \varepsilon^{-1} \left(\dfrac{\nu_2}{\nu_1} + 1\right) & (S) \end{cases}$$

(S) = SPD case

(N) = Non-symmetric case