Remark 1.27:

For the best spectral equivalence constants

$$\gamma_1 = \lambda_{min}(C_h^{-1}K_h) \quad \text{and} \quad \gamma_2 = \lambda_{max}(C_h^{-1}K_h)$$

we obtain

$$q_{opt} = \frac{\kappa(C_h^{-1}K_h) - 1}{\kappa(C_h^{-1}K_h) + 1} .$$

HENCE: Good preconditioners C_h:

1) $\kappa(C_h^{-1}K_h) \searrow 1$
2) $\text{ops}(C_h^{-1} * \underline{d}_h) = O(N_h)$

are required!

INDEED, in our example (see Sect. 1.4.2):

$$\underbrace{c_1 h}_{=\gamma_1} I_h \le K_h \le \underbrace{c_2 h^{-1}}_{=\gamma_2} I_h ,$$

the unpreconditioned case ($C_h = I_h$) = classical RICHARDSON gives

$$\frac{\gamma_2}{\gamma_1} = O(h^{-2}) \quad !$$

$$\Rightarrow \quad \boxed{I(\varepsilon) = O(h^{-1} \ln \varepsilon^{-1})}$$

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Remark 1.28