- <u>Symmetric Case:</u>

$a(\cdot,\cdot)$ symmetric $\Longrightarrow$ $K = K^T$ symmetric

In the symmetric case conditions $(46)_{C_h}$ and $(47)_{C_h}$ simplify to the following spectral equivalence inequalities:

$$(49) \quad \nu_1 C_h \le K_h \le \nu_2 C_h ,$$

yielding better bounds for the convergence rate:
$\longrightarrow$ see also NuAn!

<u>Theorem 1.26:</u>

Ass.: Let C_h and K_h be SPD matrices satisfying the spectral equivalence inequalities (49).

St.: Then the iterative method (48) is q-linear convergent with
$q(\tau) := \max\{|1-\tau\nu_1|, |1-\tau\nu_2|\} < 1$
for $\tau \in (0, 2/\nu_2)$:

$$\|\underline{u}_h - \underline{u}_h^{n+1}\|_{\substack{C_h \\ K_h \\ K_h C_h^{-1} K_h}} \le q(\tau)\, \|\underline{u}_h - \underline{u}_h^{n}\|_{\substack{C_h \\ K_h \\ K_h C_h^{-1} K_h}}$$

The optimal (minimal) rate

$$q_{opt} = q(\tau_{opt}) = \frac{\nu_2 - \nu_1}{\nu_2 + \nu_1} = \frac{(\nu_2/\nu_1) - 1}{(\nu_2/\nu_1) + 1}$$

is attained at $\tau_{opt} = 2/(\nu_1 + \nu_2)$.

<u>Proof:</u> mns or see NuAn. $\square$ q.e.d.