

Implementation of the FE - Assembling Algorithm:

Computation of $K_h^{(k)} = \begin{bmatrix} * & * \\ * & * \end{bmatrix}$, $f_h^{(k)} = \begin{bmatrix} * \\ * \end{bmatrix}$ and assembling to $\tilde{K}_h = [\tilde{K}_{ij}]_{(N_h+1) \times (N_h+1)}$, $\tilde{f}_h \in \mathbb{R}^{N_h+1}$;

Initiate: $\tilde{f} := \mathbb{0}$, $\tilde{K} := \mathbb{0}$

FOR $k=1$ STEP 1 UNTIL N_h DO (Loop over all elements)

FOR $\alpha=0$ STEP 1 UNTIL 1 DO

BEGIN

* compute $f_\alpha^{(k)} := \frac{h_\alpha}{2} f(F_K(\xi_\alpha))$

* determine $i := i(K, \alpha)$ Connectivity $K: i \leftrightarrow \alpha$

* update $\tilde{f}_i := \tilde{f}_i + f_\alpha^{(k)}$

FOR $\beta=0$ STEP 1 UNTIL 1 DO

BEGIN

* compute $K_{\alpha\beta}^{(k)} := \begin{cases} 1/h_\alpha & \text{for } \alpha=\beta \\ -1/h_\alpha & \text{for } \alpha \neq \beta \end{cases}$

* determine $j := j(K, \beta)$

* update $\tilde{K}_{ij} := \tilde{K}_{ij} + K_{\alpha\beta}^{(k)}$

$K: i \leftrightarrow \beta$

END

ENDFOR

END

ENDFOR

ENDFOR

$\Rightarrow \tilde{f}_h, \tilde{K}_h$ (without BC which will be incorporated in an extra loop over the boundary!)

$\Rightarrow$ Dirichlet: $x=0: u_0 = g_0 \leadsto \tilde{f}_1 := \tilde{f}_1 + g_0/h_1$
Cancellation of the 0. row and column!

Robin: $x=N+1: -u(1) = \alpha_1(u(1) - g_1)$

$\leadsto \tilde{K}_{N,N} := \tilde{K}_{N,N} + \alpha_1, \tilde{f}_N := \tilde{f}_N + \alpha_1 g_1$

$\Rightarrow \tilde{f}_h \in \mathbb{R}^{N_h}, \tilde{K}_h \in \mathbb{R}^{N_h \times N_h}$:

$$\tilde{K}_h \tilde{u}_h = \tilde{f}_h$$