Theorem 4.1: Let $s > 3/2$.
(a) Assume that the weak solution $u$ of (4), i.e. the solution of $(4)_{VF}$ \( \exists ! \), belongs to $H^s(T_h)$. Then $u$ satisfies the DG identity (4).
(b) Conversely, if $u \in H^s(\Omega) \cap H^s(T_h)$ satisfies the DG identity (4), then $u$ is also the solution of our VP $(4)_{VF}$.

Proof: (a) mms $\checkmark$ (b) mms$^*$

DG-Scheme:
Let us define the DG-space $V_k(T_h)$:
\[ V_k(T_h) := \{ v \in L^2(\Omega) : v|_t \in P_k(t) \text{ for } t \in T_h \} \subset H^s(T_h) \]
Then the DG scheme reads as follows:

\[ (4)_h \]
\[ \text{Find } u_h = u_{0h} \in V_k(T_h) : \]
\[ a_h(u_h, v_h) = (f, v_h) \quad \forall v_h \in V_k(T_h) \]

\[ (4)_h \]
\[ \text{Find } u_h \in \mathbb{R}^n : K_h u_h = f_h \text{ in } \Omega_h \]

Remark 4.2:
1. The Dirichlet BC $u = 0$ on $\Gamma$ is incorporated in (4) resp. $(4)_h$! (mms)
2. $\beta = -1$: SIPG = Symmetric Interior Penalty Galerkin
$\beta = +1$: NIPG = Non-symmetric IPG
$\beta = 0$: IIPG = Incomplete IPG
3. $\exists! u_h$: $(4)_h$ ? L & M ?
YES: Show $V_k(T_h)$-ell. & $-T$ of $Q_h(\cdot, \cdot)$ w.r.t. DG norm!