b) **Approximation Estimate:** \[ \| u \|_{W^2_2(\omega)} \leq \frac{C(u)}{h} \| u \|_{W^2_2(\omega)} \]

by mapping to a reference domain, application of Bramble-Hilbert's Lemma, and return mapping: Under the assumption

- (i) \( u \in W^2_2(\Omega) \),
- \( \Omega \) - regular grid, i.e. \( \mathcal{T}_\Delta \) - regular triangulation,
- and additional smoothness requirements imposed on the data \( \{ a, c, f \} \),

we can prove the estimate

\[ |(\mathcal{N}, \mathcal{Z})| \leq C(u) h \| u \|_{W^2_2(\omega)} \]

Indeed, from the splitting of the approximation error

\[ \Psi(x) = L_h u - \bar{f} = 0 \]

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