3.4.2. Discrete Convergence of the classical Integral Balance Method

For the DS (6) PB (similarly, for (6) n = 0, ...)

\[ v = u_h : \omega_h \rightarrow \Omega^h: L_h v(x) = f_h(x), \ x \in \omega \]

\[ L_h v(x) = g_h(x), \ x \in \Gamma_h = \partial \Omega \]

\[ v(x) = g_1(x), \ x \in \Gamma_1 = \partial \Omega_1 \]

(6)

Error estimates in discrete norms follow from the STABILITY and the APPROXIMATION with respect to the corresponding norms:

1. **Discrete Convergence in the \( W^4_{2,0}(\omega_h) \)-norm:**

\[ \| u - v \|_{W^4_{2,0}(\omega_h)} \leq C(u) \]

where \( W^4_{2,0}(\omega_h) = H^4(\omega) := \{ z: \omega \rightarrow \mathbb{R}^4; \ z|_{\Gamma_h} = 0 \} \) with the norm

\[ \| z \|_{W^4_{2,0}(\omega)} = \sqrt{\sum_{x \in \Gamma_h} \zeta^2 \frac{2}{h} (x)^2} H^4(x) + \sum_{x \in \Gamma_h} \zeta^2 \frac{2}{h} \frac{1}{H^4(x)} + \sum_{x \in \Gamma_h} \zeta^2 \frac{1}{H^4(x)} \]

with \( \zeta^2 \frac{2}{h} (x)^2 = \left( \frac{2}{h} (x) - 2(x) \right) \frac{1}{h (x, \Omega)} \)

H(\Omega) = S(\Omega) h(\Omega, \Omega)

(iii) uniform grid: \( \bigcirc \bigcirc \bigcirc \)

(iv) locally non-uniform grids: uniformity is only perturbed for 20 \( O(h^{-1}) \) triangles (along the boundary, interface)