E 2.5 Show that a regular family of triangular meshes in the sense of definition (4) is also a regular triangulation in the sense of Definition 2.3! Provide the constants $e_1, e_2, e_3$ and $c_3$!

E 2.6 Compute $\lambda_{\text{min}}(G_0) =$ ? and $\lambda_{\text{max}}(G_1) =$ ? for Courant's element:

\[
\begin{align*}
p^{(3)}(\bar{\xi}) &= \bar{\xi}_1, \\
p^{(2)}(\bar{\xi}) &= \bar{\xi}_2, \\
p^{(0)}(\bar{\xi}) &= 1 - \bar{\xi}_1 - \bar{\xi}_2
\end{align*}
\]

and $\bar{\xi} =$ ? and $\bar{\Gamma} =$ ? for our model problem (2) on a regular triangular mesh! 

Hint: Use the results of E 2.5!

E 2.7 Show that the eigenvalue estimates (11) are sharp with respect to the h-order, i.e.

\[\exists \xi_E, \xi'_E = \text{const} > 0: \xi_E' \leq \xi_E h^d \quad \text{and} \quad \lambda_{\text{min}}(K_h) \leq \xi'_E h^{-2} \quad \text{and} \quad \lambda_{\text{max}}(K_h) \geq \xi'_E h^{-2} \]

Therefore: $\lambda_{\text{min}} = O(h^d), \lambda_{\text{max}} = O(h^{-2}), \text{se}(K_h) = O(h^{-2})$.

E 2.8 Show the spectral equivalence inequalities

\[\varepsilon_h h^d (u_h, u_h) \leq (M_h u_h, u_h) = \| u_h \|^2_{L^2(\Omega)} \leq \varepsilon_h h^d (u_h, u_h) \]

$\forall u_h = \{ u^{(i)} \}_{i \in \omega_h} \leftrightarrow u_h = \sum_{i \in \omega_h} u^{(i)} p^{(i)} \in V_h$, 

with the mass matrix

\[M_h = \left[ \int_{\Omega} p^{(j)} p^{(i)} \right]_{i, j \in \omega_h} \]