Fluid Mechanics: \((\rightarrow)\) Lectures "Math Mod"

0) Stationary Navier-Stokes Equations:

Describing the stationary flow of an incompressible Newtonian fluid \((d=3)\):

\[ \frac{1}{Re} \Delta u + (u \cdot \nabla) u + \nabla p = f \text{ in } \Omega \subset \mathbb{R}^3, \]

\[ \text{div } u = 0 \text{ in } \Omega, \]

\[ + \text{BC: e.g. } u = 0 \text{ on } \Gamma = \partial \Omega. \]

(22) \_CF

\[ \text{Find the velocity field } u(x) = (u_1(x), u_2(x), u_3(x))^T \]

and the pressure field \(p(x)\):

\[ \frac{1}{Re} \Delta u + (u \cdot \nabla) u + \nabla p = f \text{ in } \Omega \subset \mathbb{R}^3, \]

\[ \text{non-linear convection term} \]

\[ \text{div } u = 0 \text{ in } \Omega, \]

\[ + \text{BC: e.g. } u = 0 \text{ on } \Gamma = \partial \Omega. \]

\[ \text{VF } \]

\[ \text{Re} = \frac{\rho}{\mu} \lVert \nu \rVert = \frac{1}{\nu} \lVert \nu \rVert - \text{dimensionless} \]

Reynolds number

(22) \_VF

\[ \text{Find } u \in V := (H^1(\Omega))^3 \text{ and } p \in Q = \{ q \in L^2(\Omega): \int_{\Omega} q \, dx = 0 \} \]

\[ a(u; u, v) + b(v, p) = \langle F, v \rangle \quad \forall v \in V \]

\[ b(u, q) = \langle G, q \rangle \quad \forall q \in Q \]

where \(a(\cdot; \cdot, \cdot): \mathbb{V} \times \mathbb{V} \times \mathbb{V} \rightarrow \mathbb{R}^1 \)-cont. trilinear form,

\[ a(w_i; u, v) = \frac{1}{Re} \int_{\Omega} \nabla u \cdot \nabla v \, dx + \int_{\Omega} \sum_{j=1}^{3} \sum_{k=1}^{3} w_{ij} \frac{\partial u_j}{\partial x_k} v_k \, dx, \]

\[ b(u, q) = \int_{\Omega} \text{div } u \cdot q \, dx - \text{continuous bilinear form}, \]

\[ \langle F, v \rangle := \int_{\Omega} f T v \, dx, \quad G = 0 \]

Due to the convection term the N-S-problem (22)

is non-linear. Solvability investigation \((\exists + !)\) is more difficult! The fix-point linearization leads to the so-called Ossen problem!