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2. Elementsteifigkeitmatrizen $K^{(r)} = \left[K^{(r)}_{\alpha\beta} \right]_{\alpha,\beta \in A}$:

$$K^{(r)}_{\alpha\beta} = \int_{\delta_r} \Big[\lambda_1(x) \frac{\partial \varphi^{(r)}_\beta(x)}{\partial x_1} \frac{\partial \varphi^{(r)}_\alpha(x)}{\partial x_1} + \lambda_2(x) \frac{\partial \varphi^{(r)}_\beta(x)}{\partial x_2} \frac{\partial \varphi^{(r)}_\alpha(x)}{\partial x_2} +$$

$$+ a(x)\, \varphi^{(r)}_\beta(x)\, \varphi^{(r)}_\alpha(x) \Big]\, dx$$

$$\delta_r \to \Delta \qquad \frac{\partial}{\partial x_1} = \frac{\partial}{\partial \xi_1} \frac{\partial \xi_1}{\partial x_1} + \frac{\partial}{\partial \xi_2} \frac{\partial \xi_2}{\partial x_1} = \ldots\ldots$$

$$= \int_\Delta \Big[\lambda_1(x_{\delta_r}(\xi)) \left(\frac{\partial \Phi_\beta}{\partial \xi_1} \frac{\partial \xi_1}{\partial x_1} + \frac{\partial \Phi_\beta}{\partial \xi_2} \frac{\partial \xi_2}{\partial x_1} \right) \left(\frac{\partial \Phi_\alpha}{\partial \xi_1} \frac{\partial \xi_1}{\partial x_1} + \frac{\partial \Phi_\alpha}{\partial \xi_2} \frac{\partial \xi_2}{\partial x_1} \right) +$$

$$+ \lambda_2(x_{\delta_r}(\xi)) \left(\frac{\partial \Phi_\beta}{\partial \xi_1} \frac{\partial \xi_1}{\partial x_2} + \frac{\partial \Phi_\beta}{\partial \xi_2} \frac{\partial \xi_2}{\partial x_2} \right) \left(\frac{\partial \Phi_\alpha}{\partial \xi_1} \frac{\partial \xi_1}{\partial x_2} + \frac{\partial \Phi_\alpha}{\partial \xi_2} \frac{\partial \xi_2}{\partial x_2} \right) +$$

$$+ a(x_{\delta_r}(\xi))\, \Phi_\beta(x)\, \Phi_\alpha(x) \Big]\, |J_{\delta_r}|\, d\xi$$

$$\approx \Big[\qquad\qquad \Big]_{\xi = \xi_*} |J_{\delta_r}| \underbrace{\text{meas}\,\Delta}_{= 1/2}$$

 $\xi_* = \left(\frac{1}{3}, \frac{1}{3}\right)$

mit (im Falle linearer Formfunktionen)

$$\Phi_1(\xi) = 1 - \xi_1 - \xi_2 \qquad \frac{\partial \Phi_1}{\partial \xi_1} = -1 \qquad \frac{\partial \Phi_1}{\partial \xi_2} = -1$$

$$\Phi_2(\xi) = \xi_1 \qquad \frac{\partial \Phi_2}{\partial \xi_1} = 1 \qquad \frac{\partial \Phi_2}{\partial \xi_2} = 0$$

$$\Phi_3(\xi) = \xi_2 \qquad \frac{\partial \Phi_3}{\partial \xi_1} = 0 \qquad \frac{\partial \Phi_3}{\partial \xi_2} = 1$$

$$\boxed{K^{(r)}_{\alpha,\beta} := \Big[-\text{"}- \Big]_{\xi = \xi_*} |J_{\delta_r}| \cdot \frac{1}{2}}$$

Ausgeschriebene Formeln siehe Skriptum!