c) Verallgemeinerungen: $[0,1] \rightsquigarrow [a,b]$

1) Abbildung $[a,b]$ auf $[0,1]$: $\Longleftarrow$ a), b)

$$u(x) = u(x(\xi))$$

$$\downarrow$$

$$x(\xi) = a + (b-a)\xi$$

2) Zerlegen $[a,b] = \bigcup_{i=1}^{n} [x_{i-1}, x_i] = \bigcup_{i=1}^{n} \delta_i$

$$\downarrow$$

$$[x_{i-1}, x_i] \overset{\xi = \xi_{\delta_i}(x)}{\underset{}{\rightleftarrows}} [0,1] \leftarrow \text{a), b)}$$

$$x = x_{\delta_i}(\xi) = (x_i - x_{i-1})\xi + x_{i-1}$$

$$dx = (x_i - x_{i-1})\,d\xi = h\,d\xi$$

Bsp. Integration

$$\int_a^b u(x)\,dx = \sum_{i=1}^{n} \int_{x_{i-1}}^{x_i} u(x)\,dx$$

$$= \sum_{i=1}^{n} (x_i - x_{i-1}) \int_0^1 \overset{b)}{u}\big((x_i - x_{i-1})\xi + x_{i-1}\big)\,d\xi$$

$$\overset{b)}{\approx} \sum_{i=1}^{n} (x_i - x_{i-1}) \sum_{\alpha=1}^{p+1} u\big((x_i - x_{i-1})\xi_\alpha + x_{i-1}\big)\, w_\alpha$$

Verallgemeinerte Newton-Cotes Formeln

p = 1: Verallgemeinerte Trapezregel

$$\int_a^b u(x)\,dx \approx \frac{h}{2}\big(u(x_0) + 2u(x_1) + \ldots + 2u(x_{n-1}) + u(x_n)\big)$$

p = 2: Verallgemeinerte Keplersche Faßregel = Simpson-Regel

$$\int_a^b u(x)\,dx \approx \frac{h}{6}\Big[u(x_0) + 4u(x_{1/2}) + 2u(x_1) + 4u(x_{1+\frac{1}{2}}) + 2u(x_2) + \\ + 2u(x_{n-1}) + 4u(x_{n-1+\frac{1}{2}}) + u(x_n)\Big]$$

wobei $x_{i+\frac{1}{2}} = x_i + \frac{1}{2}(x_{i+1} - x_i) = x_i + \frac{h}{2}$.