

Folglich gilt:

$$\int_{x_{i-1}}^{x_i} [z'(x)]^2 dx = \int_{x_{i-1}}^{x_i} \left[z'(x) - \underbrace{\frac{1}{h} \int_{x_{i-1}}^{x_i} z'(\xi) d\xi}_{=0} \right]^2 dx$$

$$= \int_{x_{i-1}}^{x_i} \left[\underbrace{z'(x) - \frac{1}{h} \int_{x_{i-1}}^{x_i} z'(\xi) d\xi}_{= z(x_i) - z(x_{i-1}) = 0 - 0 = 0} \right]^2 dx$$

$$= \int_{x_{i-1}}^{x_i} \left[\frac{1}{h} \int_{x_{i-1}}^{x_i} (z'(x) - z'(\xi)) d\xi \right]^2 dx$$

$$= \int_{x_{i-1}}^{x_i} \left[\int_{\xi}^x z''(\eta) d\eta \right]^2 dx$$

$$= \int_{x_{i-1}}^{x_i} \left[\frac{1}{h} \int_{x_{i-1}}^{x_i} \int_{\xi}^x z''(\eta) d\eta d\xi \right]^2 dx$$

$$\leq \int_{x_{i-1}}^{x_i} \left\{ \frac{1}{h^2} \int_{x_{i-1}}^{x_i} 1^2 d\xi \int_{x_{i-1}}^{x_i} \left(\int_{\xi}^x z''(\eta) d\eta \right)^2 d\xi \right\} dx$$

Cauchy

$$= \int_{x_{i-1}}^{x_i} \left\{ \frac{1}{h^2} \underbrace{(x_i - x_{i-1})}_{=h} \int_{x_{i-1}}^{x_i} \left(\int_{\xi}^x 1 \cdot z''(\eta) d\eta \right)^2 d\xi \right\} dx$$

Cauchy \rightarrow 11

$$\left| \int_{\xi}^x 1^2 d\eta \right| \cdot \left| \int_{\xi}^x (z''(\eta))^2 d\eta \right|$$

$$\underbrace{\int_{x_{i-1}}^{x_i} 1^2 d\eta}_{=x_i - x_{i-1} = h} \cdot \int_{x_{i-1}}^{x_i} (z''(\eta))^2 d\eta$$

$$\leq \int_{x_{i-1}}^{x_i} \frac{1}{h^2} \cdot h \int_{x_{i-1}}^{x_i} h \int_{x_{i-1}}^{x_i} (z''(\eta))^2 d\eta d\xi dx$$

$$= \int_{x_{i-1}}^{x_i} dx \int_{x_{i-1}}^{x_i} d\xi \int_{x_{i-1}}^{x_i} (z''(\eta))^2 d\eta$$

$$= h \cdot h \int_{x_{i-1}}^{x_i} (u''(\eta) - \underbrace{\tilde{u}_h''(\eta)}_{=0})^2 d\eta$$

$$= h^2 \int_{x_{i-1}}^{x_i} (u''(\eta))^2 d\eta$$