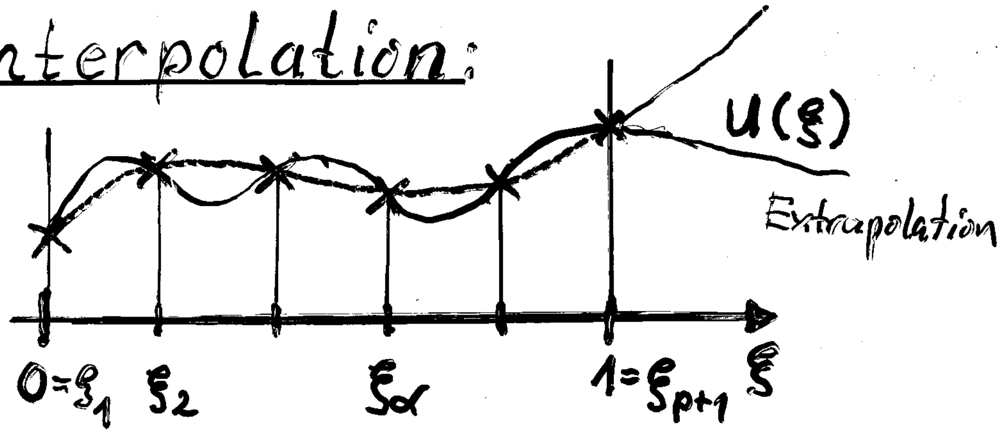
Bem.: zur Anwendung der Lagrangen Interpolationspolynome

a) Interpolation:



$$\tilde{u}(\xi) := \sum_{\alpha=1}^{p+1} \overset{\times}{u}_\alpha \, \Phi_\alpha(\xi) \approx u(\xi)$$

$$\overset{\times}{u}_\alpha = u(\xi_\alpha)$$

b) Numerische Integration: Newton-Cotes-Formeln

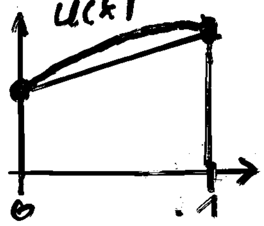
$$\int_0^1 u(\xi)\,d\xi \approx \int_0^1 \tilde{u}(\xi)\,d\xi = \sum_{\alpha=1}^{p+1} u(\xi_\alpha) \underbrace{\int_0^1 \Phi_\alpha(\xi)\,d\xi}_{=:\, w_\alpha}$$

$$= \sum_{\alpha=1}^{p+1} u(\xi_\alpha) \; w_\alpha$$

w_α: Gewichte

ξ_α: Stützstellen

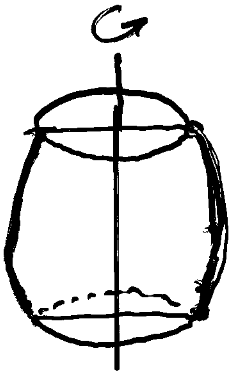
Bsp.1 $p=1$ Trapezregel



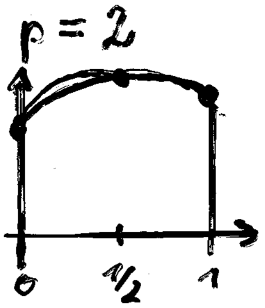
$\xi_1 = 0: \quad w_1 = \int_0^1 (1-\xi)\,d\xi = 1/2$

$\xi_2 = 1: \quad w_2 = \int_0^1 \xi \, d\xi = 1/2$

$\int_0^1 u(\xi)\,d\xi \approx \frac{1}{2}\left(u(0) + u(1)\right)$



$p=2$ Keplersche Faßregel!



$\xi_1 = 0: \quad w_1 = \int_0^1 (2\xi^2 - 3\xi + 1)\,d\xi = \frac{1}{6}$

$\xi_2 = 1/2: \quad w_2 = \int_0^1 (-4\xi^2 + 4\xi)\,d\xi = \frac{4}{6}$

$\xi_3 = 1: \quad w_3 = \int_0^1 (2\xi^2 - \xi)\,d\xi = \frac{1}{6}$

$\int_0^1 u(\xi)\,d\xi \approx \frac{1}{6}\left(u(0) + 4u(\tfrac{1}{2}) + u(1)\right)$