Folglich gilt:

$$\int_{x_{i-1}}^{x_i} [z'(x)]^2 dx = \int_{x_{i-1}}^{x_i} \Big[z'(x) - \overbrace{\frac{1}{h} \underbrace{\int_{x_{i-1}}^{x_i} z'(\xi) d\xi}_{= z(x_i) - z(x_{i-1}) = 0 - 0 = 0}}^{=0} \Big]^2 dx$$

$z'(x) = \frac{1}{h} \int_{x_{i-1}}^{x_i} z'(x) d\xi$

$$= \int_{x_{i-1}}^{x_i} \Big[\frac{1}{h} \int_{x_{i-1}}^{x_i} \underbrace{(z'(x) - z'(\xi))}_{= \int_{\xi}^{x} z''(\eta) d\eta} d\xi \Big]^2 dx$$

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$$= \int_{x_{i-1}}^{x_i} \Big[\frac{1}{h} \int_{x_{i-1}}^{x_i} 1 \cdot \int_{\xi}^{x} z''(\eta) d\eta \, d\xi \Big]^2 dx$$

Cauchy

$$\leq \int_{x_{i-1}}^{x_i} \Big\{ \frac{1}{h^2} \int_{x_{i-1}}^{x_i} 1^2 d\xi \int_{x_{i-1}}^{x_i} \Big(\int_{\xi}^{x} z''(\eta) d\eta \Big)^2 d\xi \Big\} dx$$

$$= \int_{x_{i-1}}^{x_i} \Big\{ \frac{1}{h^2} \underbrace{(x_i - x_{i-1})}_{=h} \int_{x_{i-1}}^{x_i} \underbrace{\Big(\int_{\xi}^{x} 1 \cdot z''(\eta) d\eta \Big)^2}_{\text{Cauchy} \to \ \leq} d\xi \Big\} dx$$

$$\Big| \int_{\xi}^{x} 1^2 d\eta \Big| \cdot \Big| \int_{\xi}^{x} (z''(\eta))^2 d\eta \Big|$$

$$\leq \underbrace{\int_{x_{i-1}}^{x_i} 1^2 d\eta}_{= x_i - x_{i-1} = h} \cdot \int_{x_{i-1}}^{x_i} (z''(\eta))^2 d\eta$$

$$\leq \int_{x_{i-1}}^{x_i} \frac{1}{h^2} \cdot h \int_{x_{i-1}}^{x_i} h \int_{x_{i-1}}^{x_i} (z''(\eta))^2 d\eta \, d\xi \, dx$$

$$= \int_{x_{i-1}}^{x_i} dx \int_{x_{i-1}}^{x_i} d\xi \int_{x_{i-1}}^{x_i} (z''(\eta))^2 d\eta$$

$$= h \cdot h \int_{x_{i-1}}^{x_i} (u''(\eta) - \underbrace{\tilde{u}_h''(\eta)}_{=0})^2 d\eta$$

$$= h^2 \int_{x_{i-1}}^{x_i} (u''(\eta))^2 d\eta$$