

- Der Raum $H^1(\Omega) \equiv W_2^1(\Omega)$: (Sobolev-Räume)

$$H^1(\Omega) := \{u \in L_2(\Omega) : \exists \frac{\partial u}{\partial x_i} \in L_2(\Omega), i=1, \dots, d\}$$

$$\|u\|_{H^1(\Omega)}^2 \equiv \|u\|_{1,\Omega}^2 := \|u\|_{0,\Omega}^2 + \|u\|_{0,\Omega}^2 = \int_{\Omega} (|u|^2 + |\nabla u|^2) dx,$$

$$(u,v)_{H^1(\Omega)} \equiv (u,v)_{1,\Omega} := \int_{\Omega} \{uv + \nabla^T u \nabla v\} dx,$$

$$\nabla u = \left(\frac{\partial u}{\partial x_1}, \dots, \frac{\partial u}{\partial x_d} \right)^T, |\nabla u|^2 = \left(\frac{\partial u}{\partial x_1} \right)^2 + \dots + \left(\frac{\partial u}{\partial x_d} \right)^2,$$

Cauchy-Ungl.: $|(u,v)_1| \leq \|u\|_1 \|v\|_1 \quad \forall u,v \in H^1(\Omega)$

- Die Sobolev-Räume $H^K(\Omega) \equiv W_2^K(\Omega)$:

$$H^K(\Omega) := \{u \in L_2(\Omega) : \exists \partial^\alpha u \in L_2(\Omega) \quad \forall |\alpha| \leq K\}$$

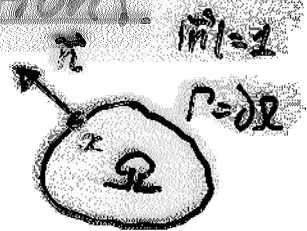
$$\|u\|_{H^K(\Omega)} = \|u\|_{K,\Omega} \equiv \|u\|_K := (u,u)_K^{0.5}$$

$$(u,v)_K \equiv (u,v)_{K,\Omega} \equiv (u,v)_{H^K(\Omega)} := \sum_{|\alpha| \leq K} \int_{\Omega} \partial^\alpha u \partial^\alpha v dx$$

- Formeln der partiellen Integration:

- Grundformel: $\vec{n} = \vec{n}(x) = (n_1(x), \dots, n_d(x))^T$

$$(2) \quad \int_{\Omega} \frac{\partial w}{\partial x_i} dx = \int_{\partial\Omega} w(x) n_i(x) ds_x$$



- Folgerungen:

a) Formel der partiellen Integration: $w = u \cdot v$

$$(2)a) \quad \int_{\Omega} \frac{\partial u}{\partial x_i} \cdot v dx = - \int_{\Omega} u \frac{\partial v}{\partial x_i} dx + \int_{\partial\Omega} u \cdot v \cdot n_i ds_x$$

b) Gauß: $w = w_i, \quad \sum (\operatorname{div} \vec{w} = \sum_{i=1}^d \frac{\partial w_i}{\partial x_i})$

$$(2)b) \quad \int_{\Omega} \operatorname{div} \vec{w} dx = \int_{\partial\Omega} \vec{w}^T \vec{n} ds_x$$

c) Green: $u \mapsto \frac{\partial u}{\partial x_i}, \quad \sum m \cdot a)$

$$(2)c) \quad \int_{\Omega} \Delta u \cdot v dx = - \sum_{i=1}^d \int_{\Omega} \frac{\partial u}{\partial x_i} \frac{\partial v}{\partial x_i} dx + \int_{\partial\Omega} \frac{\partial u}{\partial n} \cdot v ds_x = 1$$

$$= \int_{\Omega} u \cdot \Delta v dx - \int_{\partial\Omega} u \frac{\partial v}{\partial n} ds_x + \int_{\partial\Omega} \frac{\partial u}{\partial n} v ds_x \quad 2.$$