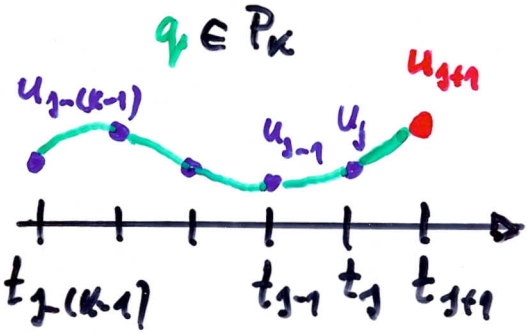
5.5.5. BDF - Verfahren

- Ausgangspkt. = ①
- Idee:

$$u'(t_{j+1}) = f(t_{j+1}, u(t_{j+1}))$$



Interpolation von $u(\cdot)$ mit
- Stützstellen: t_i
- Stützwerten: u_i ($\approx u(t_i)$)

$i = n-(j-1), \dots, j-1, j, j+1$

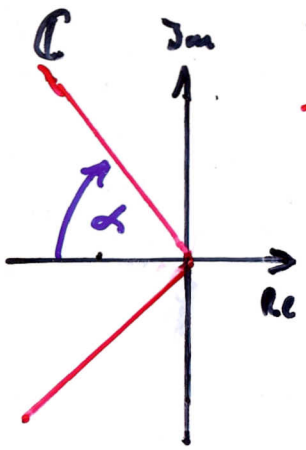
$$q'(t_{j+1}) \approx f_{j+1} := f(t_{j+1}, u_{j+1})$$

- BDF = Backward Difference Formula ($K = 1, 2, \dots$)

$$\sum_{i=0}^{K} \delta_i \nabla^i u_{j+1} = \tau f_{j+1} \quad \text{mit } \delta_0 = 0,\ \delta_i = \frac{1}{i},\ i \geq 1,$$

$$\text{und } \nabla^{j+1} = \nabla^j \cdot \nabla,\ \nabla^0 = I,\ \nabla u_{j+1} = u_{j+1} - u_j.$$

- Bem.:
  1. BDF sind implizite MSV
  2. $K0 = K$, 0-stabil nur für $K \leq 6$ | $A(\alpha)$-stabil
  3. $K=1$: $u_{j+1} - u_j = \tau f_{j+1}$ — $A(90°) = A$



→ $K=2$: $\frac{3}{2}u_{j+1} - 2u_j + \frac{1}{2}u_{j-1} = \tau f_{j+1}$ ← — $A(90°) = A$

$K=3$: $\frac{11}{6}u_{j+1} - 3u_j + \frac{3}{2}u_{j-1} - \frac{1}{3}u_{j-2} = \tau f_{j+1}$ — $A(86{,}03°)$

$K=4$: $\frac{25}{12}u_{j+1} - 4u_j + 3u_{j-1} - \frac{4}{3}u_{j-2} + \frac{1}{4}u_{j-3} = \tau f_{j+1}$ — $A(73{,}35°)$

$K=5$: $\frac{137}{60}u_{j+1} - 5u_j + 5u_{j-1} - \frac{10}{3}u_{j-2} + \frac{5}{4}u_{j-3} - \frac{1}{5}u_{j-4} = \tau f_{j+1}$ — $A(51{,}84°)$

$K=6$: $\frac{49}{20}u_{j+1} - 6u_j + \frac{15}{2}u_{j-2} - \frac{20}{3}u_{j-3} + \frac{15}{4}u_{j-4} - \frac{6}{5}u_{j-5} + \frac{1}{6}u_{j-6} = \tau f_{j+1}$ — $A(17{,}84°)$