

■ Bemerkung zur Abschätzung (19): + or -

- Btr. Ü4: Bsp. 4.2 (N-2):

$$u'(t) = -50(u(t) - \cos(\kappa\pi t)) =: f(t, u(t))$$

$$u'(t) = \lambda u(t) + g(t) =: f(t, u(t))$$

- Abschätzung des globalen Fehlers für expl. Euler:

$$\begin{aligned} |u(t_j) - u_j| &= |u(t_j) - (u(t_{j-1}) + \tau \lambda u(t_{j-1}) + \tau g(t_{j-1})) \\ &\quad + (u(t_{j-1}) + \tau \lambda u(t_{j-1}) + \tau g(t_{j-1})) - (u_{j-1} + \tau(\lambda u_{j-1} + g(t_{j-1})))| \\ &= |d_\tau(t_j) + (u(t_{j-1}) - u_{j-1}) + \tau \lambda (u(t_{j-1}) - u_{j-1})| \\ &= |d_\tau(t_j) + (1 + \tau \lambda)(u(t_{j-1}) - u_{j-1})| \leq \\ &\leq |d_\tau(t_j)| + \underbrace{|1 + \tau \lambda|}_{\leq 1!} |u(t_{j-1}) - u_{j-1}| \end{aligned}$$

1) $\lambda = +\alpha = L > 0 \Rightarrow |1 + \tau \lambda| = 1 + \tau \alpha > 1!$

2) $\lambda = -\alpha = -50 < 0 \Rightarrow |1 - \tau \alpha| \leq 1$

z.B.

$$\Leftrightarrow 1 - \tau \alpha \geq -1$$

$$\Leftrightarrow \boxed{\tau \leq 2/\alpha}$$

$$\leq \underbrace{|d_\tau(t_j)|}_{\leq C_K(u) \tau^2} + |u(t_{j-1}) - u_{j-1}| \leq \dots \leq$$

$$\leq C_K(u) \tau^2 \sum_{l=0}^{j-1} 1 \leq m + |u(t_0) - u_0|$$

$$\leq C_K(u) \tau (\tau \cdot m) + |u(t_0) - u_0| \quad \text{Rdf} = 0$$

$$= C_K(u) T \tau + |u(t_0) - u_0| \approx C_K(u) T \tau$$