Die Voraussetzungen (V0)–(V2) sind für unser Bsp. (4) aus PÜ. 2.2 erfüllt. Tatsächlich,

(V0) $|\langle F, v\rangle| = |\int_a^b f(x)v(x)dx + \alpha_b g_b v(b)| \leq$

$$\leq \|f\|_0 \|v\|_0 + |\alpha_b||g_b|\,|v(b)|$$

NR: $v(b) = \int_a^b v'(x)dx = \int_a^b v'(x)1dx$ $(v(a)=0)$

$$|v(b)| = |\int_a^b v'(x)\cdot 1 dx| \leq \sqrt{\int_a^b (v'(x))^2 dx}\sqrt{\int_a^b 1^2 dx}$$

$$\leq \sqrt{b-a}\,\|v\|_1$$

$\|v\|_0 \leq \|v\|_1$ $\leq \|f\|_0\|v\|_1 + |\alpha_b||g_b|\sqrt{b-a}\,\|v\|_1 = c\|v\|_1$

mit $c = \|f\|_0 + |\alpha_b||g_b|\sqrt{b-a}$.

(V1) $a(v,v) = \int_a^b v'v'dx + \underbrace{\alpha_b}_{\geq 0} v^2(b) \geq \int_a^b (v'(x))^2 dx =$

$$= \frac{1}{2}\int_a^b (v'(x))^2 dx + \frac{1}{2}\int_a^b (v'(x))^2 dx \geq$$

$$\geq \frac{1}{2}\frac{1}{c_F^2}\int_a^b (v(x))^2 dx + \frac{1}{2}\int_a^b (v')^2 dx \geq \mu_1 \|v\|_1^2$$

PÜ. 2.1

Friedrichs Ungl.: $\int_a^b v^2 dx \leq c_F^2 \int_a^b (v')^2 dx \;\forall v \in \bar{V}_0$; $\mu_1 = \min\{\frac{1}{2c_F^2}, \frac{1}{2}\}$

(V2) $|a(w,v)| = |\int_a^b w'v'dx + \alpha_b w(b)v(b)| \leq$

$$\leq |\int_a^b w'(x)v'(x)dx| + |\alpha_b||w(b)|\,|v(b)|$$

Cauchy

$$\leq \sqrt{\int_a^b (w'(x))^2 dx}\sqrt{\int_a^b (v'(x))^2 dx} + \alpha_b |w(b)||v(b)|$$

NR $\leq \|w\|_1\|v\|_1 + \alpha_b(b-a)\|w\|_1\|v\|_1$

$$= \underbrace{(1+\alpha_b(b-a))}_{=:\,\mu_2}\|w\|_1\|v\|_1 \quad \forall w,v \in \bar{V}_0$$