c) Verallgemeinerungen: $[0,1] \rightsquigarrow [a,b]$

1) Abbildung $[a,b]$ auf $[0,1]$: $\leftarrow$ a), b)

$$u(x) = u(x(\xi))$$
$$\downarrow$$
$$x(\xi) = a + (b-a)\xi$$

2) Zerlegen $[a,b] = \bigcup_{i=1}^{n} [x_{i-1}, x_i] = \bigcup_{i=1}^{n} \delta_i$

$$\downarrow$$

$$[x_{i-1}, x_i] \underset{\longleftarrow}{\xrightarrow{\xi = \xi_{\delta_i}(x)}} [0,1] \leftarrow \text{a), b)}$$

$x = x_{\delta_i}(\xi) =$

$= (x_i - x_{i-1})\xi + x_{i-1}$

$dx = (x_i - x_{i-1}) d\xi = h\, d\xi$

Bsp. Integration

$$\int_a^b u(x)dx = \sum_{i=1}^{n} \int_{x_{i-1}}^{x_i} u(x)\,dx$$

$$= \sum_{i=1}^{n} (x_i - x_{i-1}) \int_0^1 u((x_i - x_{i-1})\xi + x_{i-1})\, d\xi \quad \text{b)}$$

$$\overset{\text{b)}}{\approx} \sum_{i=1}^{n} (x_i - x_{i-1}) \sum_{\alpha=1}^{p+1} u((x_i - x_{i-1})\xi_\alpha + x_{i-1})\, W_\alpha$$

Verallgemeinerte Newton – Cotes Formeln

$p=1$: Verallgemeinerte Trapezregel

$$\int_a^b u(x)dx \approx \frac{h}{2}\left(u(x_0) + 2u(x_1) + \ldots + 2u(x_{n-1}) + u(x_n)\right)$$

$p=2$: Verallgemeinerte Keplersche Faßregel

= Simpson - Regel

$$\int_a^b u(x)dx \approx \frac{h}{6}\Big[u(x_0) + 4u(x_{1/2}) + 2u(x_1) + 4u(x_{1+\frac{1}{2}}) + 2u(x_2) + \ldots + 2u(x_{n-1}) + 4u(x_{n-1+\frac{1}{2}}) + u(x_n)\Big]$$

wobei $x_{i+\frac{1}{2}} = x_i + \frac{1}{2}(x_{i+1} - x_i) = x_i + \frac{h}{2}$.