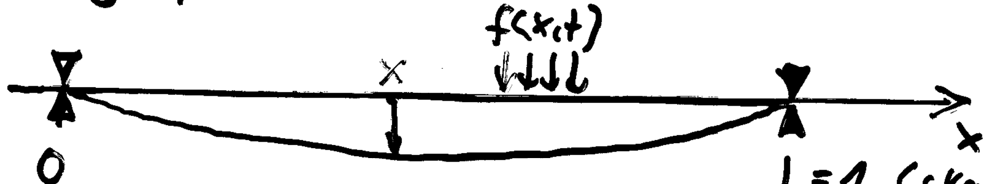
1.2.2. Schwingungsgleichung

- Die Modellierung der Schwingungen einer fest eingespannten Saite



Auslenkung: $u(x,t)$ (klein!!)

führt auf das <u>mathematische Modell (=ARWA)</u>:

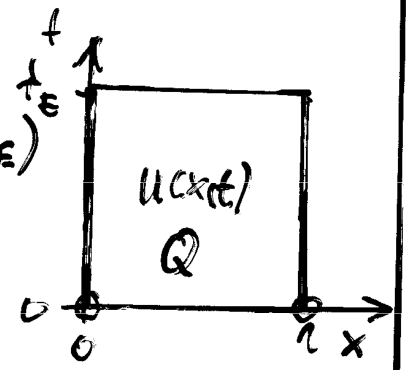
(7) Ges. $u = u(x,t) \in C^2(Q) \cap C(\overline{Q}) \cap C^1(\underline{Q})$:

$$\frac{\partial^2 u}{\partial t^2}(x,t) - a^2 \frac{\partial^2 u}{\partial x^2}(x,t) = f(x,t) \quad \forall (x,t) \in Q = (0,1) \times (0,t_E)$$

RB: 1. Art (Dirichlet) $\curvearrowright$ 1. ARWA

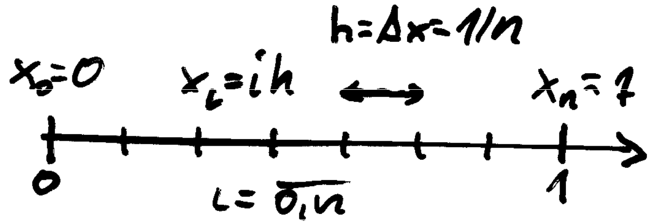
$$u(0,t) = u(1,t) = 0 \quad \forall t \in (0,t_E)$$

AB: $\left.\begin{aligned} u(x,0) &= u_0(x) \\ \frac{\partial u}{\partial t}(x,0) &= u_1(x) \end{aligned}\right\} \forall x \in [0,1]$

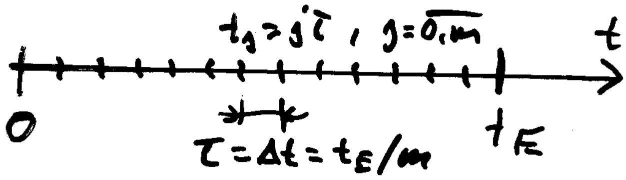


- Analog zur Diskretisierung des Wärmeleitproblems ersetzen wir

$[0,1] \ni x \mapsto x_i = ih$:



$[0,t_E] \ni t \mapsto t_j = j\tau$:



$$\frac{\partial^2 u}{\partial x^2}(x_i,t_j) \approx \frac{u(x_{i+1},t_j) - 2u(x_i,t_j) + u(x_{i-1},t_j)}{h^2}$$

$$\frac{\partial^2 u}{\partial t^2}(x_i,t_j) \approx \frac{u(x_i,t_{j+1}) - 2u(x_i,t_j) + u(x_i,t_{j-1})}{\tau^2}$$