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Talk announcement

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Space-time Finite Element Methods

We derive space-time finite element methods for parabolic evolution problems on completely unstructured decompositions of the space-time cylinder $Q \subset R^{d+1}$ into$(d+1)$-dimensional simplices. Our approach treats the time t as just another variable $x_{d+1} = t$which allows for full space-time adaptivity as well as fullspace-time parallelization that is different from the popular parallel-in-time methods. Starting from the maximal parabolic regularity setting, which holds under natural assumptions on the data, we derive consistent finite element schemes that are coercive on the conforming finite element space. We show discretization error estimates for smooth and non-smooth solutions in a mesh-dependent norm. While the maximal parabolic regularity setting holds for many important applications, it excludes problems where the right hand side maycontain a distributional term that comes from the spatial divergence of a L_2 vector-field. If this vector-field is piecewise smooth, then we are able to derive coercive and consistent finite element schemes,together with a priori discretization error estimates. We perform some a posteriori error analysis, and review a posteriori error indicators from the literature that fit to full space-time adaptivity. Finally, we apply the space-time approach to a PDE-constrained optimization problem with standard L_2-regularization. The resulting finite element method for solving the reduced first-order optimality system is again coercive on the finite element space, and we can show a priori discretization error estimates. In order to fully utilize the potential of the space-time technique, we consider adaptive versions. We derive new functional a posteriori error estimators that provide guaranteed upper bounds on the discretization error and can be used to guide adaptive refinements. In addition to the a priori and a posteriori error analysis we present an extensive set of numerical examples and benchmark problems, to support the theoretical findings. We also study the (parallel) performance of existing black-box AMG preconditioners.