Robust Additive Multilevel Methods by Algebraic Multigrid

Hans Regler
Institut für Informatik
Technische Universität München
D-80290 München

Michael Griebel
Institut für Angewandte Mathematik
Abteilung Wissenschaftliches Rechnen und Numerische Simulation
Universität Bonn
Wegelerstraße 6
53115 Bonn

email:griebel@iam.uni-bonn.de, regler@informatik.tu-muenchen.de

Abstract

For Laplace's problem and related strong elliptic operators, multiplicative and additive multilevel methods are known to possess convergence rates which are independent of the mesh width or the number of levels used. In this sense, multilevel algorithms are optimal. Furthermore, in certain cases (MIMD–systems with slow communication), additive methods promise advantages with respect to parallelization. However, standard additive methods (i.e. the BPX-preconditioner [1]) are not robust at all, which means that the corresponding condition numbers and the convergence rates are depending on the coefficient functions of the differential operator. This renders standard multilevel methods useless for practical problems which strongly deviate from the Laplacian.

To this end, multiplicative multilevel methods have been shown to be robust if an appropriate smoother (ILU etc.) and matrix–dependent coarsening is used (see for example Reusken [2, 4]). Here, the basic idea is that, for singular perturbed problems, the smoother becomes an exact solver in the limit. However, this approach is not applicable for additive methods since, here, for the case of an exact solver on all levels, the resulting condition number is exactly equal to the number of levels and the method looses its level-independent convergence property. Therefore, a robust and

optimal additive method can only be achieved by maintaining point-wise smoothers like Jacobi or Gauss-Seidel and using an appropriate construction for coarse grid operators which takes care of the remaining error frequencies. Here, the algebraic multigrid algorithm of Ruge and Stüben [3] is a good candidate for forming the respective coarser grids, the intergrid transfer operators and the coarse grid operators used in an additive multilevel method.

We explain the construction of a robust additive preconditioner and discuss its properties. Furthermore, we present results from numerical experiments for different model problems, explain how the method can be parallelized and show first run time results for the parallel version of the method.

References

[1] J. Bramble, J. Pasciak, and J. Xu, *Parallel multilevel preconditioners*, Math. Comp., 31 (1990), pp. 333–390.

[2] Reusken, A. (1994). Multigrid with matrix-dependent transfer operators for convection-diffusion problems. In P. Hemker & P. Wesseling (Eds.), *Multigrid Methods IV* (pp. 269–280).: Birkhäuser.

[3] J. Ruge and K. Stüben, *Algebraic multigrid (AMG)*, Arbeitspapiere der GMD, 210 (1986).

[4] Wittum, G. (1989). On the robustnes of ILU-smoothing. *SIAM J. Sci. Stat. Comput.*, 10, 699–717.

[5] Grauschopf, T., Griebel, M., & Regler, H. (1996). *Additive Multilevel-Preconditioners Based on Bilinear Interpolation, Matrix-Dependent Geometric Coarsening and Algebraic Multigrid Coarsening for Second Order Elliptic PDEs.* TU München, Institut für Informatik, SFB-Report 342/02/96 A.