Improved Separators by Multigrid Methods

K. Gärtner, IIS, ETH Zürich

Abstract:
The graph partioning problem has a lot of different applications. One is the partitioning problem in parallel computations. The combinatorial problem is known to be NP hard and has the following form:
Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ denote a graph, with the set of vertices $\mathcal{V}$, edges $\mathcal{E}$ and numbers of elements $v = |\mathcal{V}|$, $e = |\mathcal{E}|$.
One asks for two subsets $\mathcal{V}_1$, $\mathcal{V}_2$, $\mathcal{V} = \mathcal{V}_1 \cup \mathcal{V}_\in$ with
a) the number of edges connecting $\mathcal{V}_1$, $\mathcal{V}_2$ minimal, and
b) $|v_1 - v_2| \to \min$.

The simplest analytical analog is a Neumann eigenvalue problem – more general analogs have the following form:

$$\tilde{\mathcal{L}}_\alpha(u, \lambda) := \frac{\|\nabla u\|^2}{\|h(u)\|^2} + \lambda_2 \frac{f_\alpha^2(u)}{\|h(u)\|^2} \to \min . \tag{1}$$

Here $\|\nabla u\|^2/\|h(u)\|^2$ is the Lagrange function, $\|\nabla u\|^2$ is a suitable norm of the gradient in analogy to a) and $f_\alpha(u)$ represents an approximation of the side condition b) replaced by

$$f(u) = \int_{\text{supp } u^+} dV - \int_{\text{supp } u^-} dV = 0, \tag{2}$$

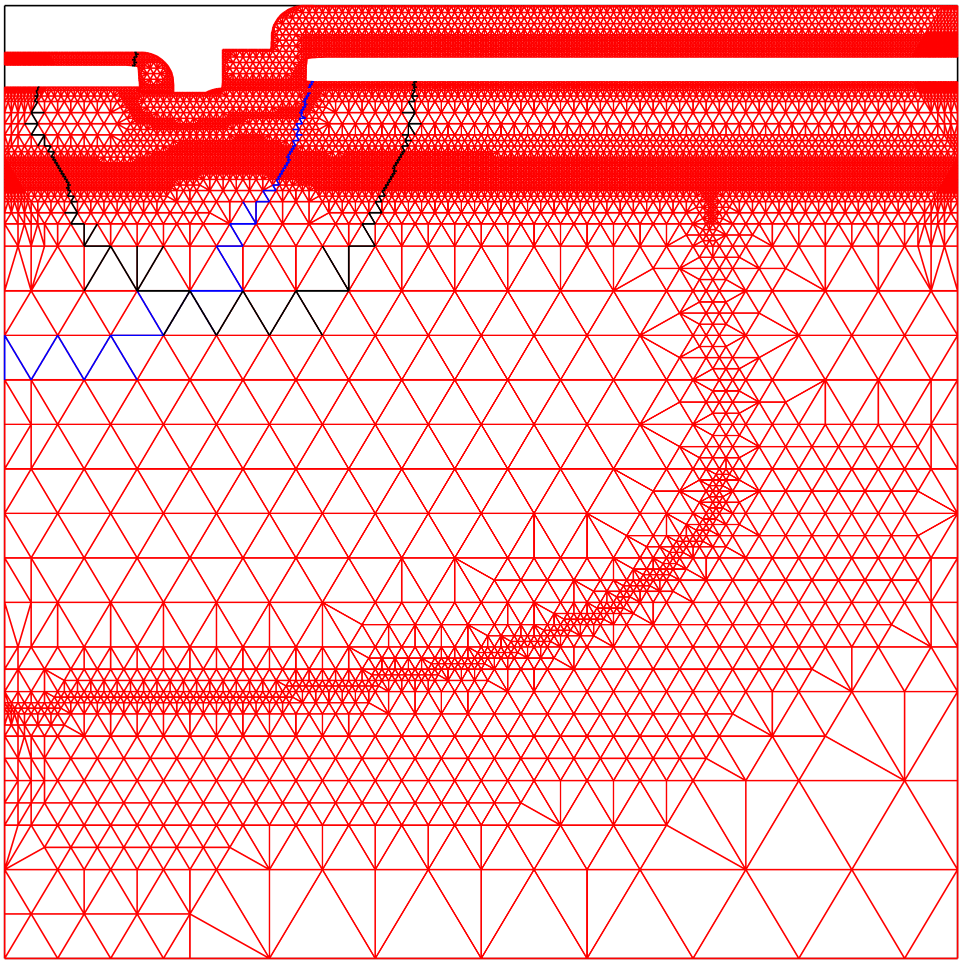
where supp u^+, supp u^- means the support of the positive / negative part of u.
Using the L_2–norm of the gradient and relaxing (2) to

$$\tilde{f}(u) = \int u dV = 0$$

yields the Neumann eigenvalue problem. In this case b) has to be approximated afterwards. This done by an ordering and counting procedure or finding a constant c that fulfills

$$\int_{\text{supp } (u+c)^+} dV - \int_{\text{supp } (u+c)^-} dV = 0.$$

Approximating the L_1–norm of the gradient and the signum function by f_α yields a family of problems requiring the solution of symmetric indefinite problems and such ones with strong variations in the coefficients. This makes the problem attractive for testing an algebraic MG algorithm. Still at the begin of exploring different possibilities some (non symmetric – the Neumann eigenvalue problem is a symmetry indicator) examples show the potential of improvements.



Separator due to the Neumann eigenvalue problem (black, 143 cut edges) and the best one (blue, 80 cut edges, known until now).