Road Map for Preparing the Oral Examination

1. Derive the variational formulation of the second-order boundary value problem (5)

$$\begin{aligned} -\mathrm{div}(A\nabla u) + b\cdot\nabla u + cu &= f \text{ in } \Omega, \\ u &= g_1 \text{ on } \Gamma_1 \\ \frac{\partial u}{\partial N} := (A\,\nabla u)\cdot n &= g_2 \text{ on } \Gamma_2, \\ \frac{\partial u}{\partial N} + \alpha u &= g_3 \text{ on } \Gamma_3, \end{aligned}$$

   given in Subsection 1.2.1, and show existence and uniqueness under some assumptions specified by the examiner !

2. Derive the variational formulation of the linear elasticity problem (8) given in Subsection 1.2.2, and show existence and uniqueness under some assumptions specified by the examiner !

3. Derive the variational formulation of the first biharmonic BVP (12) given in Subsection 1.2.3, and show existence and uniqueness of a weak solution in $H_0^2(\Omega)$! Which combinations of boundary conditions are possible ?

4. Formulate and prove Sobolev's norm equivalence theorem (Theorem 2.13) ! Use it for proving Friedrichs- and Poincaré-type inequalities !

5. Formulate and prove the Lemma of Bramble and Hilbert (Lemma 2.17) !

6. Derive the formula of integration by parts and other famous integrations formula from the main formula of the differential and integral calculus in an appropriate Sobolev space setting ! How can you use the formula of integration by parts to derive trace theorem using the example of $H(div)$ (Theorem 2.19) !

7. What do you know about mesh generation, regular meshes, and internal representation of the mesh with appropriate files ? Provide a mesh and its internal representation of some sample domains given by the examiner ! Give the definition of the FE Nodal Basis and of the V_h, V_{0h}, V_{gh} for a given triangular mesh via mapping technique !

8. Describe the generation of the FE equations $K_h \underline{u}_h = \underline{f}_h$ via the three steps

   a) assembling of the load vector,

   b) assembling of the stiffness matrix,

   c) incorporating the boundary conditions !

9. Describe the properties of the system of finite element equations and estimate the spectral condition number of the stiffness matrix K_h in the SPD case !

10. Prove the approximation error estimate

$$\inf_{v_h \in V_h} |u - v_h|_{H^s(\Omega)} \leq ?$$

    for $s = 0$ or $s = 1$, where $H^0(\Omega) = L_2(\Omega)$!

11. Prove the H^1-Convergence of the FE-solutions u_h to the solution u of a V_0-elliptic and V_0-bounded elliptic BVP !

12. How can you prove an optimal L_2 convergence rate estimate for the FE-solutions u_h to the solution u of a V_0-elliptic and V_0-bounded elliptic BVP !

13. What do you know about the L_∞-convergence of FE-solutions !

14. Prove the first Lemma of Strang ! What are the typical applications of the first Lemma of Strang ?

15. Prove the second Lemma of Strang ! What are the typical applications of the second Lemma of Strang ?

16. Explain the Clément interpolation operator and show its approximation properties !

17. Derive the residual error estimator for the homogeneous Dirichlet BVP for the Poisson equation ! How do you have to modify the residual error estimator in the case of mixed boundary conditions ?

18. Derive the residual error estimator for our heat conduction problem "CHIP" !

19. Derive and discuss Repin's functional a posteriori error estimator for the homogeneous Dirichlet BVP for the Poisson equation !

20. Derive dG variational formulations and dG schemes for the homogenous Dirichlet problem for the Poisson equation

$$-\Delta u = f \text{ in } \Omega \subset \mathbb{R}^2 \text{ and } u = 0 \text{ on } \Gamma = \partial\Omega. \tag{1}$$

Show that, under suitable conditions, the SIPG bilinear form $a_h(.,.)$ is $V_k(\mathcal{T}_h)$ elliptic and bounded ! How can you estimate the discretization error in the DG norm ?

21. Derive dG variational formulations and dG schemes for the CHIP problem ! Show that, under suitable conditions, the SIPG bilinear form $a_h(.,.)$ is $V_k(\mathcal{T}_h)$ elliptic and bounded ! How can you estimate the discretization error in the DG norm ?

22. Demonstrate the ideas of the classical Finite Difference Method (FDM) on rectangular grids and the more advanced Finite Volume Method (FVM) on trinagular grids for the Piosson equation $-\Delta u = f$ in $\Omega \in \mathbb{R}^2$ under Dirichlet boundary conditions $u = g$ on $\Gamma = \partial\Omega$! Show that discrete convergence follows from stability and approximation !

All Exercises given explicitly in lectures are also a subject of the Oral Examination !!!