Time Parallel Eddy-Current Solver

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Overview

1. Lecture Recap
2. Model problem
3. Time parallel

Model Problem

Find u given $u(0) \in \mathbb{R}$

$$M_h u_h'(t) + K_h u_h(t) = f_h(t) \tag{1}$$

With $K_h, M_h > 0$ and symmetric
Applying Implicit Euler yields:

$$-M_h u_h^k + (M_h + \tau_k K_h) u_h^{k+1} = \tau_k f_h^{k+1} \tag{2}$$

For simplicity a uniform τ is chosen

Big System

Let $A_h := M_h + \tau K_h$
We can rewrite the problem as linear system:

$$\begin{pmatrix} A_h & & & \\ -M_h & A_h & & \\ & \ddots & \ddots & \\ & & -M_h & A_h \end{pmatrix} \begin{pmatrix} u_h^1 \\ u_h^2 \\ \vdots \\ u_h^m \end{pmatrix} = \begin{pmatrix} \tau f_h^1 + M_h u_h^0 \\ \tau f_h^2 \\ \vdots \\ \tau f_h^m \end{pmatrix} \tag{3}$$

We call this matrix L_τ
We want to solve this iteratively in parallel

Richardson Scheme

Let D_τ be the block diagonal matrix of L_τ
Then we define a Richardson scheme:

$$x_{k+1} = x_k + \omega D_\tau^{-1}(f - L_\tau x_k) \tag{4}$$

Good behavior for $\omega \in [0.5, 1)$
$\omega = 1$ is the same as sequential solving

An appropriate choice **smooths** the error,
Easy hierarchical mesh $\rightarrow$ Multigrid Methods

Multigrid scheme

We use the Richardson Smoother in a Multigrid Method:

- Apply smoother ν_1-times
- $d = R(f - L_\tau x)$
- Solve $L_{coarse} w = d$ recursively
- $x = x + Pw$
- Apply smoother ν_2-times

Can be mostly executed in parallel, except for coarse grids

Geometry "Induction Furnace"

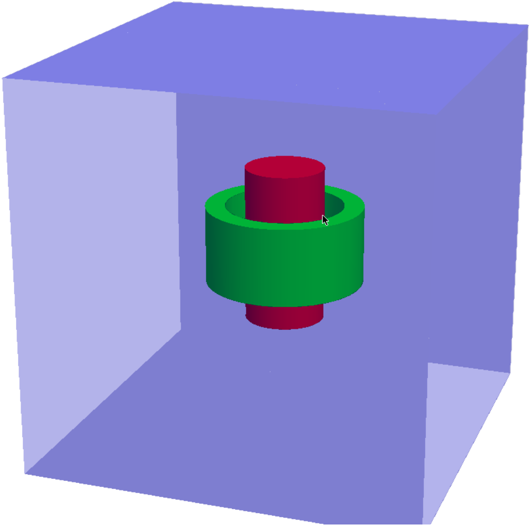

Figure: Geometry of the problem in Netgen

PDE

$$\sigma \frac{\partial E}{\partial t} + curl \mu^{-1} curl\ E = -\frac{\partial J_i}{\partial t}$$

Material parameters

- Copper Coil: $\sigma = 6 * 10^7 S/m$, $\mu = 1.2 * 10^{-6} H/m$
- Iron Core: $\sigma = 10^7 S/m$, $\mu = 6.3 * 10^{-3} H/m$
- Vacuum: $\underline{\sigma = 1 S/m}$, $\mu = 1.2 * 10^{-6} H/m$

Alternating current in copper-coil

Homogeneous Dirichlet BC

Implementation Details

- Implemented with MFEM
- Visualized with GL-Vis
- Implicit Euler scheme
- Works space-parallel
- Compatible with Neumüllers code

Surface Flux 1

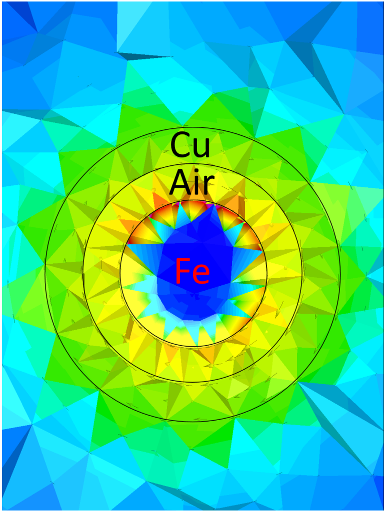

Figure: Intersection in the middle, surface fluxes visible

Surface Flux 1

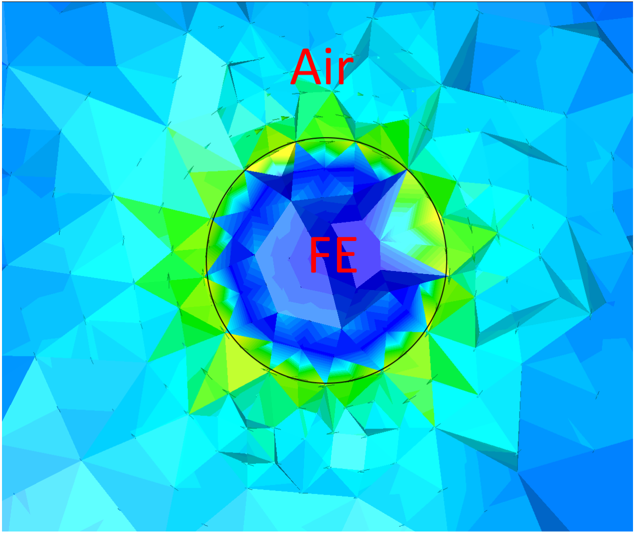

Figure: Intersection near the end of the iron core, surface fluxes still visible

Implementation

Time parallel method:

- Writing a multi-time-level system solver interface with MFEM support
  Implements A_τ and M_h
- Space Problem solved with PCG, and AMS-Preconditioner
- Integrating Neumüllers code
- Result: time & space parallel method with MPI

Used model:

- Random initial value (in space and time)
- Homogeneous BC and RHS
- Primitive mesh (technical problems)
- Two materials ("air", iron)
- Currently no visualization

Convergence & Scaling

Average convergence rate: 0.3 ± 0.05

Iterations: 18
Time Steps: 32
Degree of Freedom in Space: 13872

time / space	1	2	4	8	16	32	64
1	500.6	x	x	x	x	x	104.6
2	x	163.5	104.0	61	50.3	52.6	
4	x	83.6	52.5	32.4	28.8		
8	x	50.4	31.7	19.3			
16	x	36.2	22.0		11		
32	53.3	31.7					

Total processors is time-processors times space-processors.

Outlook

Further work will:

- Solve the "induction-furnace" problem time-parallel
- Add space coarsening for speedup
- Run tests on Vulcan
- Resolve the case $\sigma = 0$
- Couple with heat equation
- Couple with heat equation and Stokes equation

The End