Parallelization of Fluid-Structure-Interaction

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3. Juli 2017

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JKU

- Fluid-Structure Interaction
- Discretization
- Solvers
- Parallelization

Overview

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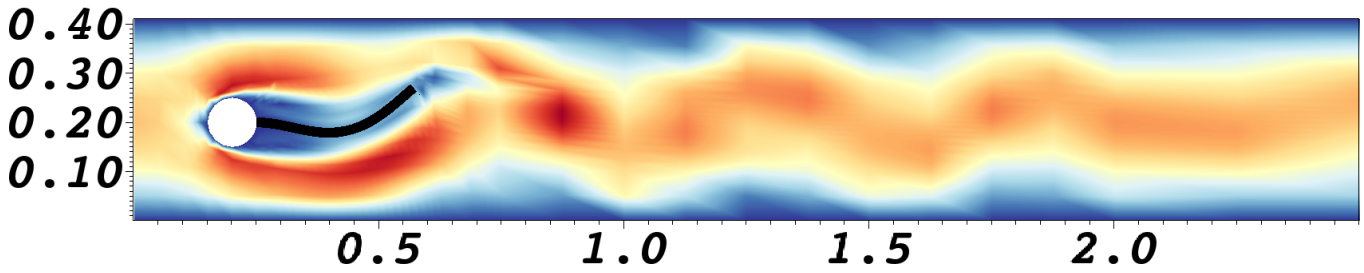
FSI

- Ingredients: fluid (Navier-Stokes), solid
- Coupling conditions:
  1. Continuity of stresses (propagate forces)
  2. Continuity of velocities (no-slip)
- ALE-coordinates
- Turek-Hron Benchmark:



FSI in ALE

$$\hat{J}\hat{\rho}_f\hat{\partial}_t\hat{v}_f + \hat{J}\hat{\rho}_f\hat{\nabla}\hat{v}_f\hat{F}^{-1}\cdot(\hat{v}_f - \partial_t\hat{\mathcal{A}}) - \mathsf{div}_R(\hat{J}\hat{\sigma}_f\hat{F}^{-T}) = 0$$
$$\hat{J}\ \mathsf{tr}(\hat{\nabla}\hat{v}_f\hat{F}^{-1}) = 0$$

$$\hat{\rho}_s\partial_t\hat{v}_s - \mathsf{div}_R(\hat{F}\hat{\Sigma}) = 0$$
$$(\partial_t\hat{u}_s - \hat{v}_s) = 0$$

$$-\alpha_u\ \mathsf{div}_R(\hat{\sigma}_{MM}(\hat{u}_f)) = 0$$

FSI in ALE

$$\hat{J}\hat{\sigma}_f\hat{F}^{-T}\hat{n}_f + \hat{F}\hat{\Sigma}_s = 0 \text{ on } \hat{\Gamma}_I \qquad \hat{v}_f = g \text{ on } \hat{\Gamma}_{in}$$

$$\hat{v}_f - \hat{v}_s = 0 \text{ on } \hat{\Gamma}_I \qquad \hat{v}_f = 0 \text{ on } \hat{\Gamma}_{tb} \cup \hat{\Gamma}_c$$

$$\hat{u}_f - \hat{u}_s = 0 \text{ on } \hat{\Gamma}_I \qquad \hat{u}_f = 0 \text{ on } \partial\hat{\Omega}_f \backslash \hat{\Gamma}_I$$

$$\hat{u}_s, \hat{v}_s = 0 \text{ on } \hat{\Gamma}_{cf}$$

- Do-nothing condition:

$$\hat{J}(-\hat{p}_f I + \hat{\rho}_f\hat{\nu}_f\hat{\nabla}\hat{v}_f\hat{F}^{-1})\hat{F}^{-T} = 0 \text{ on } \hat{\Gamma}_{out}.$$

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Discretization

1. Variational formulation
2. Time-stepping (One-step-θ-scheme)
3. Linearization (Newton's method)
4. Spatial discretization ($Q(2) - Q(2) - Q(1)$ for u,v,p)
5. Interface conditions: global elements

Coupled System

$$\mathcal{A} :=$$

$\mathcal{M}^{\Omega\Omega}$		$\mathcal{M}^{\Omega I}$				
	$\mathcal{S}_{uu}^{\Omega\Omega}$	$\mathcal{S}_{uu}^{\Omega I}$	$\mathcal{S}_{uv}^{\Omega\Omega}$	$\mathcal{S}_{uv}^{\Omega I}$		
$\mathcal{M}^{I\Omega}$	$\mathcal{S}_{uu}^{I\Omega}$	$\mathcal{M}_{uu}^{II} + \mathcal{S}_{uu}^{II}$	$\mathcal{S}_{uv}^{I\Omega}$	$\mathcal{S}_{uv}^{II}$		
	$\mathcal{S}_{vu}^{\Omega\Omega}$	$\mathcal{S}_{vu}^{\Omega I}$	$\mathcal{S}_{vv}^{\Omega\Omega}$	$\mathcal{S}_{vv}^{\Omega I}$		
$\mathcal{B}_{fm}^{I\Omega}$	$\mathcal{S}_{vu}^{I\Omega}$	$\mathcal{B}_{fm}^{II} + \mathcal{S}_{vu}^{II}$	$\mathcal{S}_{vv}^{I\Omega}$	$\mathcal{S}_{vv}^{II} + \mathcal{F}_{vv}^{II}$	$\mathcal{F}^{I\Omega}$	$\mathcal{F}_{vp}^{I}$
$\mathcal{B}_{fm}^{\Omega\Omega}$		$\mathcal{B}_{fm}^{\Omega I}$		$\mathcal{F}^{\Omega I}$	$\mathcal{F}^{\Omega\Omega}$	$\mathcal{F}_{vp}^{\Omega}$
$\mathcal{B}_{pm}^{\Omega}$		$\mathcal{B}_{pm}^{I}$		$\mathcal{F}_{pv}^{I}$	$\mathcal{F}_{pv}^{\Omega}$	0

$$:= \begin{bmatrix} \mathcal{M} & \mathcal{C}_{ms} & 0 \\ \mathcal{C}_{sm} & \mathcal{S} & \mathcal{C}_{sf} \\ \mathcal{C}_{fm} & \mathcal{C}_{fs} & \mathcal{F} \end{bmatrix}$$

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Solvers

Solvers:

- Sparse-Direct:
  - easy to use, robust
  - 2d: $\mathcal{O}(n^{\frac{3}{2}})$ flops, $\mathcal{O}(n \log n)$ memory ($\mathcal{O}(n^{\frac{4}{3}}), \mathcal{O}(n^2)$ in 3d)
- Iterative:
  - less memory requirement
  - need good preconditioner

Preconditioning

FGMRES with

- "Black-box" preconditioners (ILU, AMG, ...): not good
- Multigrid [Richter, ...]
- Approximate Block-LU-factorization, partitioned scheme: [Heil][Gee,Küttler,Wall][Langer, Yang]

Preconditioning Steps

Motivated by the block-factorization of A, we arrive at the following steps to compute P^{-1}:

1. $x_m = \mathcal{M}^{-1} r_m$
2. $x_s = \mathcal{S}^{-1} r_s$
3. $x_f = \mathcal{F}^{-1}(r_f - \mathcal{C}_{fs} x_s - \mathcal{C}_{fm} x_m)$
4. $x_s = \mathcal{S}^{-1}(\mathcal{C}_{sf} x_f)$
5. $x_m = \mathcal{M}^{-1}(\mathcal{C}_{mf} x_f)$

$\rightarrow$ Similar to partitioned solver
$\rightarrow$ Iterative solvers to approximate inverses
$\rightarrow \varnothing$ 5-25 iterations for different $h, \Delta t$ and material parameters

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Parallelization

- FEM-library deal.II
- Parallel computation with Trilinos (vectors, matrices, ...), p4est (partitioning)
- Distributed setting: each CPU only stores parts of the problem
- Communication via MPI

Triangulation

- Distribution between fluid / solid domain:
  - Split: each CPU owns parts of the fluid XOR the solid domain (but not both)

  - Shared: each CPU owns parts of the fluid AND the solid domain

Simulation

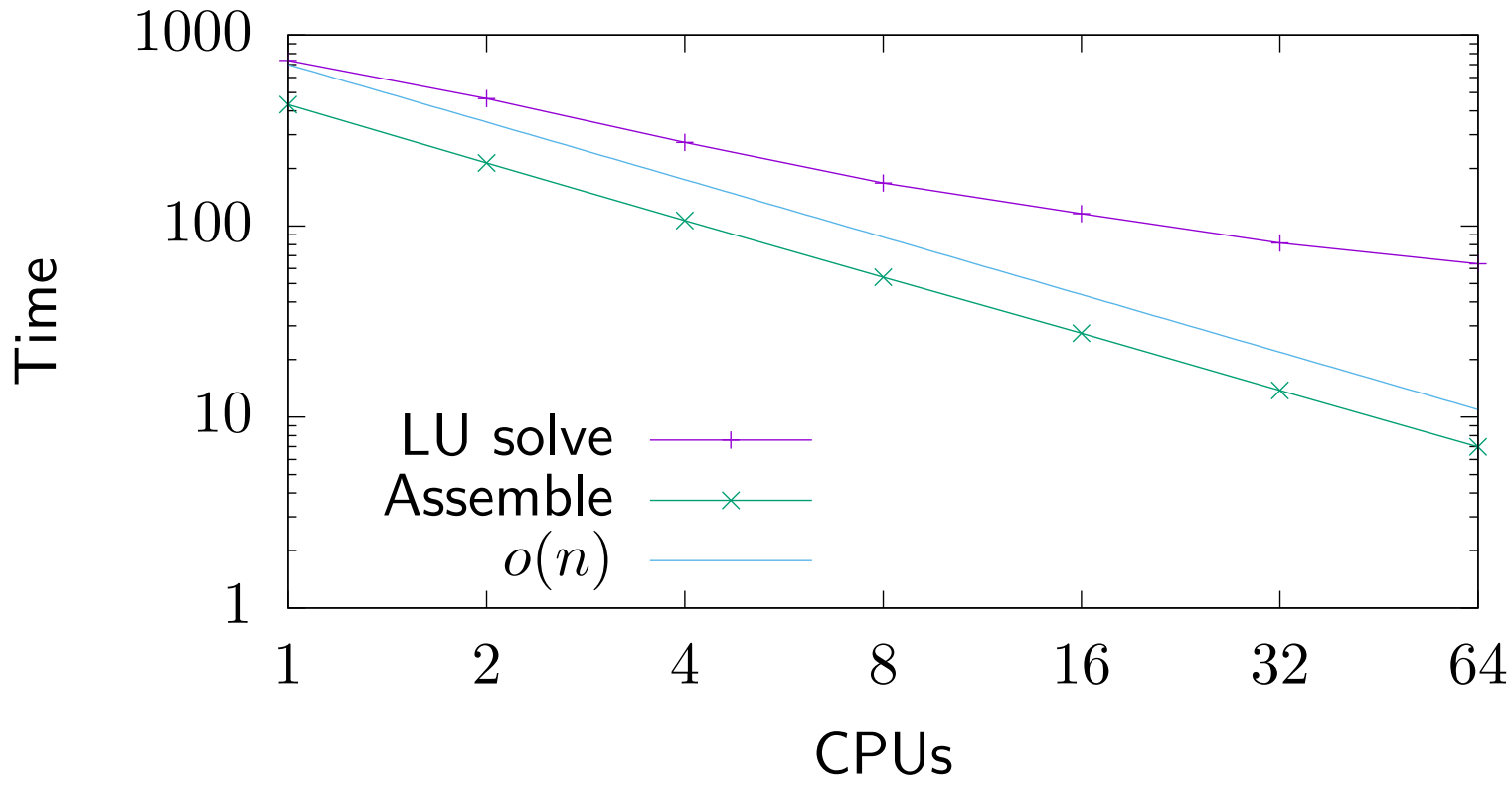
- Direct solver, $\approx 15k$ dofs

Function	Calls	%
Assemble matrix	19	63.5
Assemble rhs	62	4.03
Solve	28	31.1

But:

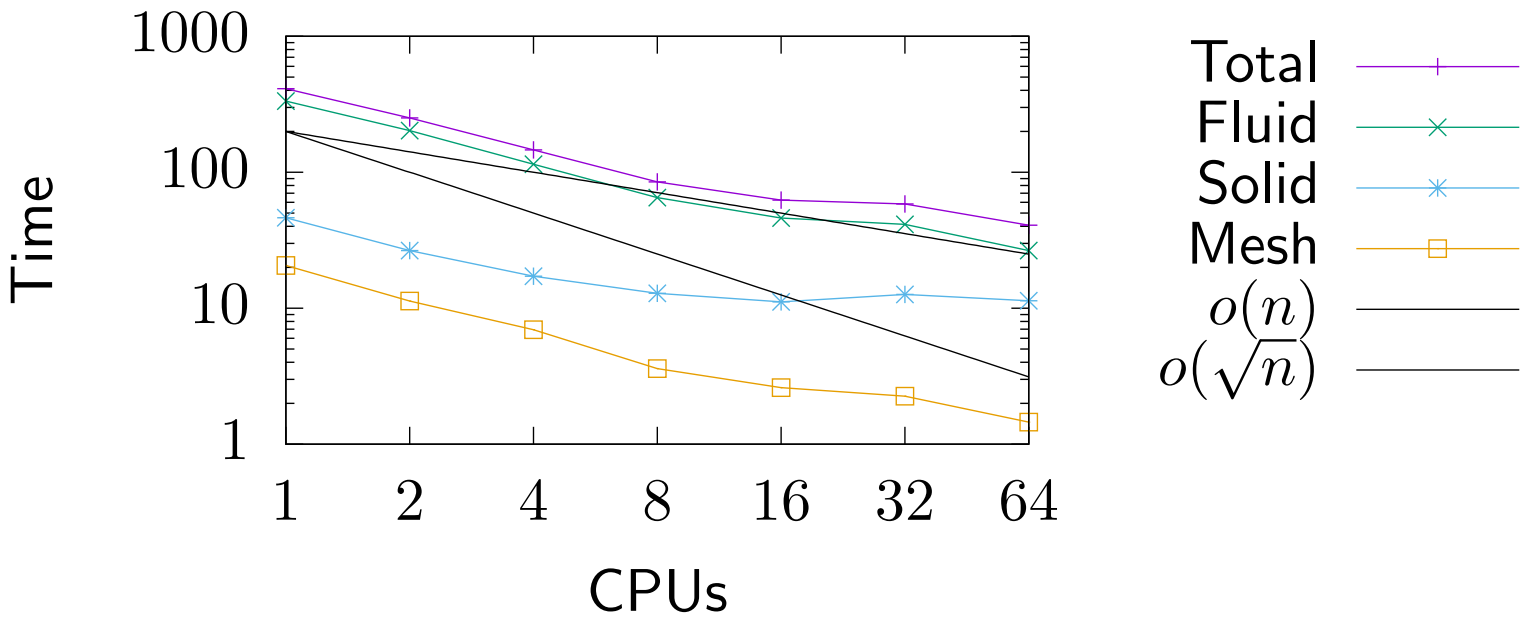
- solving time increases faster for higher refinements
- assembling scales better if more CPUs are used

Direct solver for $\approx 4m$ dofs.



Scalability of the Iterative Solver

- $\approx 15m$ dofs, shared-type partitioning
- Similar results as in [1], no other monolithic scalability tests found



[1] Crosetto, Deparis, Fourestey, Quarteroni: Parallel Algorithms for Fluid-Structure Interaction Problems in Haemodynamics

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Thank you for your attention!