# T U T O R I A L

## "Computational Mechanics"

to the lecture

"Numerical Methods in Continuum Mechanics 1"

## Tutorial 06-07

Date: Thursday, 12 May 2016
Time : $10^{\underline{15}} – 11^{\underline{45}}$
Room : K 001A

### 3.2 Solvers for Mixed Finite Element Equations

**17** Let $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{m\times n}$, and $C \in \mathbb{R}^{m\times m}$ such that $A = A^T$, $A > 0$, $C = C^T$, $C \geq 0$, and $\operatorname{Rank} B = \min\{m, n\}$ , $m \leq n$. Show that

1. the matrix $\begin{pmatrix} A & B^T \\ B & -C \end{pmatrix}$ is symmetric but indefinite, and
2. if $C > 0$, then the matrix $\begin{pmatrix} A & B^T \\ -B & C \end{pmatrix}$ is positive definite !

**18** Let $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{m\times n}$, and $C \in \mathbb{R}^{m\times m}$ such that $A = A^T$, $A > 0$, $C = C^T$, $C \geq 0$, and $\operatorname{Rank} B = \min\{m, n\}$ , $m \leq n$. Show that the dual Schur complement matrix $S = BA^{-1}B^T + C$ is symmetric and positive definite (SPD) !

**19*** Show that the best (largest) discrete LBB-constant is given by the relation

$$\tilde{\beta}_1^2 = \lambda_{min}(G_\Lambda^{-1}(BG_X^{-1}B^T)), \tag{3.31}$$

i.e. $\tilde{\beta}_1^2 = \lambda_{min}(G_\Lambda^{-1}(BG_X^{-1}B^T))$ is the minimal eigenvalue of the generalized EVP

$$BG_X^{-1}B^T\underline{\mu} = \lambda G_\Lambda \underline{\mu}, \tag{3.32}$$

where the Gram matrices $G_X$ and $G_\Lambda$ are defined by the identities

$$(G_X\underline{u}_h, \underline{v}_h)_{\mathbb{R}^n} = (u_h, v_h)_X\,, \quad \forall \underline{u}_h, \underline{v}_h \leftrightarrow u_h, v_h \in X_h \quad \text{and}$$

$$(G_\Lambda\underline{\lambda}_h, \underline{\mu}_h)_{\mathbb{R}^m} = (\lambda_h, \mu_h)_\Lambda\,, \quad \forall \underline{\lambda}_h, \underline{\mu}_h \leftrightarrow \lambda_h, \mu_h \in \Lambda_h, \quad \text{respectively.}$$

*Hint:* Use the Rayleigh-quotient representation of the minimal eigenvalue of the generalized EVP (3.32) !

**20** Derive the Uzawa–CG–Method, i.e., the CG–Method for the Schur-Complement-System

$$\text{Given } \underline{f} \in \mathbb{R}^n \text{ and } \underline{g} \in \mathbb{R}^m\text{, find } \underline{\lambda} \in \mathbb{R}^m: \quad \left(BA^{-1}B^T + C\right)\underline{\lambda} = BA^{-1}\underline{f} - \underline{g}$$

in an algorithmic form !

**21** Show that the preconditioned Uzawa Algorithm (see formula (51) in Chapter 2 of the Lectures) is equivalent to the classical Uzawa Algorithm (see formula (48) in Chapter 2 of the Lectures) applied to the preconditioned system

$$\begin{aligned} A\underline{u} + B^T D^{-1/2}\underline{\mu} &= \underline{f}\,, \\ D^{-1/2}B\underline{u} - D^{-1/2}CD^{-1/2}\underline{\mu} &= D^{-1/2}\underline{g}\,. \end{aligned}$$