T U T O R I A L

"Computational Mechanics"

to the lecture

"Numerical Methods in Continuum Mechanics 1"

Tutorial 02 Thursday, April 7, 2016 (Time : $10^{\underline{15}}$ – $11^{\underline{00}}$, Room : K 001A)

1.3 Scalar Elliptic BVP of the Fourth Order.

04 Show that the first biharmonic BVP

$$u \in V_0 := H_0^2(\Omega) : \int_\Omega \Delta u(x) \Delta v(x) dx = \int_\Omega f(x)\ v(x) dx \ \forall v \in V_0 \tag{1.7}$$

has a unique solution, where $H_0^2(\Omega) := \{v \in H^2(\Omega) : u = \frac{\partial u}{\partial n} = 0 \text{ on } \Gamma\}$!

Hint: Use the Lax-Milgram theorem and the following inequality

$$\int_\Omega |\Delta v(x)|^2 dx \geq \mu_1 \|v\|^2_{H^2(\Omega)}, \tag{1.8}$$

which is valid for all $v \in H_0^2(\Omega)$.

05* Show inequality (1.8) !

06 Give the variational formulations for BVPs of the second, the third and the fourth kind mentioned in Example 1.3 !

07* Prove that the so-called Kirchhoff plate bilinear form

$$a(u,v) := \int_\Omega \left\{ \Delta u(x) \Delta v(x) + (1-\sigma) \left[2 \frac{\partial^2 u}{\partial x_1 \partial x_2} \frac{\partial^2 v}{\partial x_1 \partial x_2} - \frac{\partial^2 u}{\partial x_1^2} \frac{\partial^2 v}{\partial x_2^2} - \frac{\partial^2 u}{\partial x_2^2} \frac{\partial^2 v}{\partial x_1^2} \right] \right\} dx \tag{1.9}$$

is identical to the biharmonic bilinear form given in (1.7) in the case of the first BVP (i.e., on $H_0^2(\Omega)$), where $\sigma \in (0,1)$ is a given material parameter (Poisson-coefficient).

08 Derive the natural boundary conditions for the plate bilinear form (1.9) ?

Hint: Use Schwarz' theorem, i.e. $\frac{\partial^2 u}{\partial x_1 \partial x_2} = \frac{\partial^2 u}{\partial x_2 \partial x_1}$, and two times partial integrations !

09 Derive a mixed variational formulation for the first biharmonic boundary value problem (1.7) by introducing a new variable $w = \Delta u$!