T U T O R I A L

"Computational Mechanics"

to the lecture

"Numerical Methods in Continuum Mechanics 1"

Tutorial 11-12

Date: Thursday, 30 June 2016
Time : $10^{\underline{15}}$ – $11^{\underline{45}}$
Room : K 001A

4.2 The Hellinger-Reisner Principle

31 Show that every tensor-function $\sigma \in \mathbf{H}(div, \Omega)$ has a well-defined trace $\gamma_\Gamma \sigma := \sigma \cdot n|_\Gamma$ in $\mathbf{H}^{-1/2}(\Gamma)$ and that the inequality

$$\|\gamma_\Gamma \sigma\|_{\mathbf{H}^{-1/2}(\Gamma)} \leq c\, \|\sigma\|_{\mathbf{H}(div,\Omega)} \tag{4.41}$$

holds for all $\sigma \in \mathbf{H}(div, \Omega)$, i.e. the trace operator $\gamma_\Gamma \in L(\mathbf{H}(div, \Omega), \mathbf{H}^{-1/2}(\Gamma))$.

Hint: Use the identity (integration by parts)

$$\int_\Omega \operatorname{div}(\sigma) \cdot v\, dx = -\int_\Omega \sigma \cdot \nabla v\, dx + \int_\Gamma (\sigma \cdot n) \cdot v\, ds$$

that is valid for all smooth tensor function σ and for all smooth vector function v.

32 Let $v \in \mathbf{L}_2(\Omega)$ be a given vector function. Let $u \in \mathbf{H}^1_{0,\Gamma_u}(\Omega)$ be such that

$$\left(\varepsilon(u)\,,\, \varepsilon(w)\right)_0 = -\left(v\,,\, w\right)_0\,, \quad \forall w \in \mathbf{H}^1_{0,\Gamma_u}(\Omega)\,.$$

Show that $\tau := \varepsilon(u)$ is in $X = \mathbf{H}(div, \Omega)$, and that $\operatorname{div} \tau = v$.

33* Show that $\tau_n (= \tau n) = 0$ on Γ_t in the sense

$$\langle \tau n\,,\, w\rangle_{H^{-1/2}(\Gamma)\times H^{1/2}(\Gamma)} = 0\,, \quad \forall w \in \mathbf{H}^1_{0,\Gamma_u}(\Omega),$$

where τ is defined in Exercise **32**.