

T U T O R I A L

“Numerical Methods for the Solution of Elliptic Partial Differential Equations”

to the lecture

“Numerics of Elliptic Problems”

Tutorial 06

Tuesday, 6 May 2014, Time: 10¹⁵ – 11⁴⁵, Room: S2 120.

3.3 Mapping

[25] Show the inequality

$$\frac{1}{2} \sin \theta_r h_r^2 \leq |J_{\delta_r}| \leq \frac{\sqrt{3}}{2} h_r^2, \quad (3.14)$$

where h_r is the longest side and θ_r the smallest angle of the triangle δ_r .

3.4 Generation of systems of Finite Elements Equations

[26] Show that the integration rule

$$\int_{\Delta} f(\xi, \eta) d\xi d\eta \approx \frac{1}{2} \{ \alpha_1 f(\xi_1, \eta_1) + \alpha_2 f(\xi_2, \eta_2) + \alpha_3 f(\xi_3, \eta_3) \} \quad (3.15)$$

integrates quadratic polynomials exactly, if the the weights α_i and the integration points (ξ_i, η_i) are chosen as follows: $\alpha_1 = \alpha_2 = \alpha_3 = 1/3$ und $(\xi_1, \eta_1) = (1/2, 0)$, $(\xi_2, \eta_2) = (1/2, 1/2)$, $(\xi_3, \eta_3) = (0, 1/2)$.

Hint: cf. also Exercise 20 !

[27] Let us assume that $\mathcal{T}_h = \{\delta_r : r \in \mathbb{R}_h\}$ is a regular triangulation of the polygonally bounded Lipschitz domain $\bar{\Omega} = \cup_{r \in \mathbb{R}_h} \bar{\delta}_r \subset \mathbb{R}^2$ into triangles δ_r , and let $u \in H^2(\Omega)$. Let us now compute the integral

$$I(u) = \int_{\Omega} u(x) dx$$

by the quadrature rule

$$I_h(u) = \sum_{r \in \mathbb{R}_h} u(x_{\delta_r}(\xi^*)) |\delta_r|,$$

where $x_{\delta_r}(\cdot)$ maps the unit triangle Δ onto δ_r , and $\xi^* = (1/3, 1/3)$. Show that

$$|I(u) - I_h(u)| \leq ch^2 |u|_{H^2(\Omega)},$$

where c is some generic positive constant. Can you weaken the assumption that $u \in H^2(\Omega)$?

Hint: Use the mapping principle and the Bramble-Hilbert Lemma; cf. also Exercise 20 !