

T U T O R I A L

“Numerical Methods for the Solution of Elliptic Partial Differential Equations”

to the lecture

“Numerics of Elliptic Problems”

Tutorial 05 Tuesday, 29 April 2014, Time: 10¹⁵ – 11⁴⁵, Room: S2 120.

3 Galerkin FEM

3.1 Galerkin-Ritz-Method

22 Let us consider the variational problem: Find $u \in V_g = V_0 = H_0^1(0, 1)$:

$$\int_0^1 u'(x)v'(x)dx = \int_0^1 f(x)v(x)dx \quad \forall v \in V_0. \quad (3.13)$$

Solve this variational problem with the Galerkin-Method using the basis

$$V_{0h} = V_{0n} = \text{span}\{x(1-x), x^2(1-x), \dots, x^{n-1}(1-x)\},$$

where the right-hand side is given as $f(x) = \cos(2\pi x)$! Compute the stiffness matrix K_h analytically and solve the linear system $K_h \underline{u}_h = \underline{f}_h$ numerically using the Gaussian elimination method ! Consider n to be 2, 4, 8, 10, 50, 100 !

3.2 Net generation and refinement

23 In the lectures, we used the input file `*.net` (see Slide 10) for the input of the mesh data. Design and implement a new Algorithm, which inputs the file `coarse.net` containing a coarse triangulation and outputs the file `fine.net` containing the refinement of the coarse triangulation by dividing every triangle of the coarse mesh into 4 triangles (red refinement) !

24* How would you modify the algorithm from Exercise 26 in order to refine selected elements only ? Note that you have to ensure conformity of the triangulation by using the green refinement dividing a triangle into two triangles by bisection.