Theorem 1.27 (Banach's fixed point theorem)
Let W be a closed subset of a Banach space V and let

$$\psi : W \to W$$

be a contraction, i.e., there exists $q < 1$:

$$\|\psi(v) - \psi(w)\| \;\leq\; q\,\|v - w\| \qquad \forall v,\, w \in W.$$

Then there exists a unique element $u \in W$:

$$u \;=\; \psi(u).$$

The sequence $(u_k)_{k\in\mathbb{N}}$, given by the fixed point iteration

$$u_{k+1} \;=\; \psi(u_k)$$

converges to the solution u for arbitrary initial values $u_0 \in W$ and

$$\|u_{k+1} - u\| \;\leq\; q\,\|u_k - u\| \qquad \forall k \in \mathbb{N}_0\,.$$

Proof: see Analysis II or any basic course/book on calculus or functional analysis.