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**Algorithm 1:** Richardson's method

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$\mathbf{x}_0$  given

$$\mathbf{r}_0 = \mathbf{b} - \mathbf{A} \mathbf{x}_0$$

$$k = 0$$

repeat

$$\mathbf{p}_k = \mathbf{r}_k$$

$$\alpha_k = \tau$$

$$\mathbf{x}_{k+1} = \mathbf{x}_k + \alpha_k \mathbf{p}_k$$

$$\mathbf{r}_{k+1} = \mathbf{r}_k - \alpha_k \mathbf{A} \mathbf{p}_k = \mathbf{b} - \mathbf{A} \mathbf{x}_{k+1}$$

$$k = k + 1$$

until *stopping criterion fulfilled*, e.g.  $\|\mathbf{r}_k\|_{\ell^2} \leq \varepsilon \|\mathbf{r}_0\|_{\ell^2}$

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**Algorithm 2:** Method of steepest descent

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$\mathbf{x}_0$  given

$$\mathbf{r}_0 = \mathbf{b} - \mathbf{A} \mathbf{x}_0$$

$$k = 0$$

repeat

$$\mathbf{p}_k = \mathbf{r}_k$$

$$\alpha_k = \frac{(\mathbf{r}_k, \mathbf{p}_k)}{(\mathbf{p}_k, \mathbf{A} \mathbf{p}_k)}$$

$$\mathbf{x}_{k+1} = \mathbf{x}_k + \alpha_k \mathbf{p}_k$$

$$\mathbf{r}_{k+1} = \mathbf{r}_k - \alpha_k \mathbf{A} \mathbf{p}_k = \mathbf{b} - \mathbf{A} \mathbf{x}_{k+1}$$

$$k = k + 1$$

until *stopping criterion fulfilled*, e.g.  $\|\mathbf{r}_k\|_{\ell^2} \leq \varepsilon \|\mathbf{r}_0\|_{\ell^2}$

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**Algorithm 3:** Conjugate gradient method

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$\mathbf{x}_0$  given

$$\mathbf{r}_0 = \mathbf{b} - \mathbf{A} \mathbf{x}_0$$

$$k = 0$$

repeat

if  $k = 0$  then

$$\mathbf{p}_k = \mathbf{r}_k$$

else

$$\beta_{k-1} = -\frac{(\mathbf{r}_k, \mathbf{A} \mathbf{p}_{k-1})}{(\mathbf{p}_{k-1}, \mathbf{A} \mathbf{p}_{k-1})}$$

$$\mathbf{p}_k = \mathbf{r}_k + \beta_{k-1} \mathbf{p}_{k-1}$$

end

$$\alpha_k = \frac{(\mathbf{r}_k, \mathbf{p}_k)}{(\mathbf{p}_k, \mathbf{A} \mathbf{p}_k)}$$

$$\mathbf{x}_{k+1} = \mathbf{x}_k + \alpha_k \mathbf{p}_k$$

$$\mathbf{r}_{k+1} = \mathbf{r}_k - \alpha_k \mathbf{A} \mathbf{p}_k = \mathbf{b} - \mathbf{A} \mathbf{x}_{k+1}$$

$$k = k + 1$$

until *stopping criterion fulfilled*, e.g.  $\|\mathbf{r}_k\|_{\ell^2} \leq \varepsilon \|\mathbf{r}_0\|_{\ell^2}$

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