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$\boxed{01}$ Show that every linear second order partial differential equation

$$-\big(a(x)\,u'(x)\big)' + b(x)\,u'(x) + c(x)\,u(x) = f(x)\,, \tag{1.1}$$

with $a \in C^1(0,\,1)$ and $b,\,c \in C(0,\,1)$ can be rewritten in the form

$$\bar{a}(x)\,u''(x) + \bar{b}(x)\,u'(x) + c(x)\,u(x) = f(x)\,, \tag{1.2}$$

and find suitable functions $\bar{a} \in C^1(0,\,1)$ and $\bar{b} \in C(0,\,1)$. Show also the reverse direction.

$\boxed{02}$ Derive the variational formulations of the two following boundary value problems:

$$\text{(a)} \qquad \begin{cases} -u''(x) + u(x) &= f(x) \qquad\qquad \text{for } x \in (0,\,1) \\ u(0) &= g_0 \\ u(1) &= g_1 \end{cases}$$

$$\text{(b)} \qquad \begin{cases} -u''(x) + u(x) &= f(x) \qquad\qquad \text{for } x \in (0,\,1) \\ u(0) &= g_0 \\ u'(1) &= g_1 - \alpha_1\,u(1) \end{cases}$$

In particular, specify the spaces $V_g$, and $V_0$, the bilinear form $a(\cdot,\,\cdot)$, and the linear form $\langle F,\,\cdot\rangle$.
*Hint for (b):* Perform integration by parts as usual, substitute $u'(1)$ due to the Robin boundary condition, and collect the bilinear and linear terms accordingly.

$\boxed{03}$ Consider the boundary value problem

$$\begin{aligned} -\big(a(x)\,u'(x)\big)' &= 1 \qquad \forall x \in (0,\,1)\,, \\ u(0) &= 0\,, \\ a(1)\,u'(1) &= 0\,, \end{aligned} \tag{1.3}$$

where $a(x) = \sqrt{2x - x^2}$. Justify that $u(x) = \sqrt{2x - x^2}$ is a *classical* solution of (1.3), i. e., $u \in X := C^2(0,\,1) \cap C^1(0,\,1] \cap C[0,\,1]$. Furthermore, show that

$$\int_0^1 |u'(x)|^2\,dx = \infty\,.$$

*Note:* This example shows that $u \notin H^1(0,\,1)$, i. e., $u$ is no *weak* solution.