Let us consider the following theorem from the lecture:

THEOREM 10.1 *The identity*

$$\|E\|^2 = \|\,(I - T_J)\,(I - T_{J-1})\cdots(I - T_1)\,\|^2 = \frac{c_0}{1 + c_0}$$

holds, where $\overline{T}_k = T_k^* + T_k - T_k^*\,T_k$ *and*

$$c_0 = \sup_{\|v\|=1} \inf_{\sum_k v_k = v} \sum_{k=1}^{J} \left(T_k \overline{T}_k^{-1} T_k^*\, w_k, w_k \right)$$

with $w_k = \sum_{j=k}^{J} v_j - T_k^{-1} v_k$ *and* $\|\cdot\| = \|\cdot\|_a$ *is a norm induced by* $(\cdot,\cdot) = (\cdot,\cdot)_a$ *assuming that* $I - T_k$ *is non expansive, i.e.,* $\|I - T_k\| \le 1$.

$\boxed{23}$ Prove that if Π_k denotes the elliptic projection on V_k, i.e.,

$$\begin{aligned} \Pi_k \colon V &\to V_k, \\ v &\mapsto \Pi_k v, \end{aligned}$$

defined via the relation

$$a(\Pi_k v, w_k) = a(v, w_k) \quad \forall w_k \in V_k,$$

we have that

$$c_0 \le \sup_{\|v\|=1} \sum_{k=1}^{J} \left((\overline{T}_k^{-1} - I)(\Pi_k - \Pi_{k-1})v, (\Pi_k - \Pi_{k-1})v \right).$$