21 Consider the linear system

$$\hat{A}^{(k)}\hat{\mathbf{u}}^{(k)} = \hat{\mathbf{f}}^{(k)},$$

where $\hat{A}^{(k)}$ is the two-level hierarchical basis representation of the level-k stiffness matrix at an arbitrary level k, see Exercise 18.

(a) Construct a right-hand side $\hat{\mathbf{f}}^{(k)}$ with respect to the two-level hierarchical basis.

(b) Study the condition number of the additive and of the multiplicative two-level preconditioner assuming that $A_{11}^{(k)}$ is inverted exactly, i.e., $C_{11}^{(k)} = A_{11}^{(k)}$, for different values of k.

(c) Use an incomplete factorization in order to approximate $A_{11}^{(k)}$, i.e.,

$$C_{11}^{(k)} \approx A_{11}^{(k)} = L^{(k)}U^{(k)}.$$

and study its condition number for different values of k.

22 Use the preconditioned conjugate gradient (PCG) method with the linear algebraic multilevel iteration (AMLI) as a preconditioner to solve the linear system

$$\hat{A}^{(l)}\hat{\mathbf{u}}^{(l)} = \hat{\mathbf{f}}^{(l)},$$

associated with the finest level. Assume that the problem associated with the coarsest level is solved exactly and an ILU preconditioner is used to approximate the solutions of the linear system with $A_{11}^{(k)}$, see Exercise 21 (c).

Implement the linear AMLI V-cycle and study its convergence behavior. Choose proper stopping criteria.

Send your results **until February 10, 2013** to monika.wolfmayr@numa.uni-linz.ac.at!