We denote by

$$C\,\mathbf{x}_{(k)} = R\,\mathbf{x}_{(k-1)} + \mathbf{b}, \quad k = 1, 2, 3, ... \tag{2.1}$$

the basic iteration of an iterative method for solving the linear system

$$A\,\mathbf{x} = \mathbf{b}, \tag{2.2}$$

where A is a $N \times N$-matrix with the splitting

$$A = C - R \tag{2.3}$$

and the matrix C is assumed to be nonsingular.

$\boxed{03}$ Show that the iteration (2.1) is equivalent to the iteration

$$\mathbf{x}_{(k)} = \mathbf{x}_{(k-1)} + C^{-1}\left(\mathbf{b} - A\,\mathbf{x}_{(k-1)}\right), \quad k = 1, 2, 3, ...$$

$\boxed{04}$ Let $\mathbf{r}_{(k)} = \mathbf{b} - A\,\mathbf{x}_{(k)}$ denote the k-th residual and $\mathbf{e}_{(k)} = \mathbf{x}_{(k)} - \mathbf{x}$ denote the error of the k-th vector iterate. Show that

(i) $\mathbf{r}_{(k)} = \left(I - A\,C^{-1}\right)^k \mathbf{r}_{(0)}$,

(ii) $\mathbf{e}_{(k)} = \left(I - C^{-1}A\right)^k \mathbf{e}_{(0)}$.

$\boxed{05}$ Prove that the sequence of vectors $(\mathbf{x}_{(k)})_{k\geq 1}$ in (2.1) converges to the solution of (2.2) for any $\mathbf{x}_{(0)}$ if and only if

$$\rho\left(C^{-1}R\right) = \rho\left(I - C^{-1}A\right) < 1.$$