T U T O R I A L

"Numerical Methods for the Solution of Elliptic Partial Differential Equations"

to the lecture

"Numerics of Elliptic Problems"

Tutorial 03 Tuesday, 29 March 2012, Time: $10^{\underline{15}}$ – $11^{\underline{45}}$, Room: S2 219.

1.3 Scalar elliptic problems of the fourth order

11 Prove that the first biharmonic BVP

$$u \in V_0 := H_0^2(\Omega) : \int_\Omega \Delta u(x)\Delta v(x)dx = \int_\Omega f(x)v(x)dx \;\forall v \in V_0 \tag{1.8}$$

fulfils the assumptions of the Lax-Milgram Theorem, and provide the minimization problem that is equivalent to the variational formulation (1.8).

12* Give the variational formulations for second, third and fourth BVPs mentioned in Remark 1.6.2 of the Lectures, and discuss the existence and uniqueness of generalized solutions. Without loss of generality, consider homogenized essential BCs only.

13 For the Kirchhoff plate, the plate bilinear form

$$a(u,v) := \int_\Omega \left\{ \Delta u(x)\Delta v(x) + (1-\sigma)\left[2\frac{\partial^2 u}{\partial x_1 \partial x_2}\frac{\partial^2 v}{\partial x_1 \partial x_2} - \frac{\partial^2 u}{\partial x_1^2}\frac{\partial^2 v}{\partial x_2^2} - \frac{\partial^2 u}{\partial x_2^2}\frac{\partial^2 v}{\partial x_1^2} \right] \right\} dx \tag{1.9}$$

is only identical to the biharmonic bilinear form given in (1.8) in the case of the first BVP (i.e., on $H_0^2(\Omega)$), where $\sigma \in (0,1)$ is a given material parameter (Poisson-coefficient). Prove this statemant !

14* Which natural BCs can be imposed on the plate bilinear form (1.9) ?