Different regimes of Maxwell's equations $\nu = 1/\mu$

Full Maxwell

$$\varepsilon \frac{\partial^2 \boldsymbol{E}}{\partial t^2} + \sigma \frac{\partial \boldsymbol{E}}{\partial t} + \mathbf{curl}\,(\nu\,\mathbf{curl}\,\boldsymbol{E}) = -\frac{\partial \boldsymbol{J}_i}{\partial t} \tag{15}$$

$$\varepsilon \frac{\partial^2 \boldsymbol{A}}{\partial t^2} + \sigma \frac{\partial \boldsymbol{A}}{\partial t} + \mathbf{curl}\,(\nu\,\mathbf{curl}\,\boldsymbol{A}) = \boldsymbol{J}_i \tag{16}$$

Eddy current

$$\sigma \frac{\partial \boldsymbol{E}}{\partial t} + \mathbf{curl}\,(\nu\,\mathbf{curl}\,\boldsymbol{E}) = -\frac{\partial \boldsymbol{J}_i}{\partial t} \tag{17}$$

$$\sigma \frac{\partial \boldsymbol{A}}{\partial t} + \mathbf{curl}\,(\nu\,\mathbf{curl}\,\boldsymbol{A}) = \boldsymbol{J}_i \tag{18}$$

Implicit time discretization of (15)–(18)

$$\left(\frac{\varepsilon}{\tau^2} + \frac{\sigma}{\tau}\right)\boldsymbol{u} + \mathbf{curl}\,(\nu\,\mathbf{curl}\,\boldsymbol{u}) = \dots$$

where $\boldsymbol{u} = \boldsymbol{E}$ or $\boldsymbol{A}$, and $\varepsilon = 0$ in the eddy current case

Magnetostatics

$$\mathbf{curl}\,(\nu\,\mathbf{curl}\,\boldsymbol{A}) = \boldsymbol{J} \tag{20}$$

Time-harmonic Maxwell

$$(-\omega^2\varepsilon + i\sigma\omega)\widehat{\boldsymbol{E}} + \mathbf{curl}\,(\nu\,\mathbf{curl}\,\widehat{\boldsymbol{E}}) = -i\omega\,\widehat{\boldsymbol{J}}_i \tag{21}$$

$$(-\omega^2\varepsilon + i\sigma\omega)\widehat{\boldsymbol{A}} + \mathbf{curl}\,(\nu\,\mathbf{curl}\,\widehat{\boldsymbol{A}}) = \widehat{\boldsymbol{J}}_i \tag{22}$$

Common pattern:

$$\mathbf{curl}\,(\nu\,\mathbf{curl}\,\boldsymbol{u}) + \kappa\,\boldsymbol{u} = \boldsymbol{f}$$

Full Maxwell, implicit time discretization	$\kappa \in \mathbb{R}^+$
eddy current, implicit time discretization	$\kappa \in \mathbb{R}_0^+$
Magnetostatics	$\kappa = 0$
Time-harmonic Maxwell	$\kappa \in \mathbb{C} \setminus \mathbb{R}_0^+$
Time-harmonic eddy current ($\varepsilon = 0$)	$\kappa \in \mathbb{I}$

Conductivity regularization to avoid $\kappa = 0$ (possibly in subdomains):

$$\sigma_{\text{reg}}(x) = \begin{cases} \sigma(x) & \text{if } \sigma(x) > \sigma_0 \\ \sigma_0 & \text{else,} \end{cases}$$

where $\sigma_0 > 0$ is a small constant.

This regularization can be applied to the eddy current cases,
as well as to Magnetostatics:

$$\mathbf{curl}\,(\nu\,\mathbf{curl}\,\boldsymbol{A}) + \sigma_0 \boldsymbol{A} = \boldsymbol{J}$$

Approximation of the true solution as $\sigma_0 \to 0$