07 Derive the variational formulation (with details) of the following boundary value problem:

$$\begin{aligned} \mathbf{curl}\,(\frac{1}{\mu}\,\mathbf{curl}\,\boldsymbol{u}) + \kappa\,\boldsymbol{u} &= \boldsymbol{f} && \text{in } \Omega, \\ \boldsymbol{u}\times\boldsymbol{n} &= \boldsymbol{g}_D && \text{on } \Gamma_D, \\ \frac{1}{\mu}\,\mathbf{curl}\,\boldsymbol{u}\times\boldsymbol{n} &= \boldsymbol{g}_N && \text{on } \Gamma_N, \\ \frac{1}{\mu}\,\mathbf{curl}\,\boldsymbol{u}\times\boldsymbol{n} + \kappa\,(\boldsymbol{n}\times(\boldsymbol{u}\times\boldsymbol{n})) &= \boldsymbol{g}_R && \text{on } \Gamma_R, \end{aligned} \tag{3.1}$$

where $\Omega \subset \mathbb{R}^3$ is a bounded Lipschitz domain with boundary

$$\Gamma = \partial\Omega = \Gamma_D \cup \Gamma_N \cup \Gamma_R$$

and $\kappa \in \mathbb{R}$.

08 Consider the magnetostatic problem with magnetization $\boldsymbol{M} \neq 0$, i.e.

$$\begin{aligned} \mathbf{curl}\,\boldsymbol{H} &= \boldsymbol{J}, \\ \operatorname{div}\boldsymbol{B} &= 0, \\ \boldsymbol{B} &= \mu\boldsymbol{H} + \boldsymbol{M}. \end{aligned} \tag{3.2}$$

(a) Derive the vector potential formulation of (3.2).

(b) Assume that

$$\overline{\Omega} = \overline{\Omega}_1 \cup \overline{\Omega}_2, \qquad \Omega_1 \cap \Omega_2 = \emptyset$$

and that the permeability μ and the magnetization $\boldsymbol{M}$ are constant on Ω_1 and Ω_2. Derive the variational formulation of (a) for the boundary condition

$$\boldsymbol{H}\times\boldsymbol{n} = 0 \qquad \text{on } \partial\Omega.$$

Here, you may only assume that $\boldsymbol{A}$ is smooth in Ω_1 and Ω_2 but not across the interface !

Hint: Use integration by parts also for the term including $\boldsymbol{M}$ and use the jump conditions of Lemma 1.2 from the lecture.

09 Show that in the weak sense,

(a) $\forall\, v \in H^1(\Omega) : \mathbf{curl}\,\nabla v = 0$,

(b) $\forall\, \boldsymbol{w} \in H(\mathbf{curl}\,,\Omega) : \operatorname{div}\,\mathbf{curl}\,\boldsymbol{w} = 0$.

Hint: Use Definition 2.2 from the lecture.